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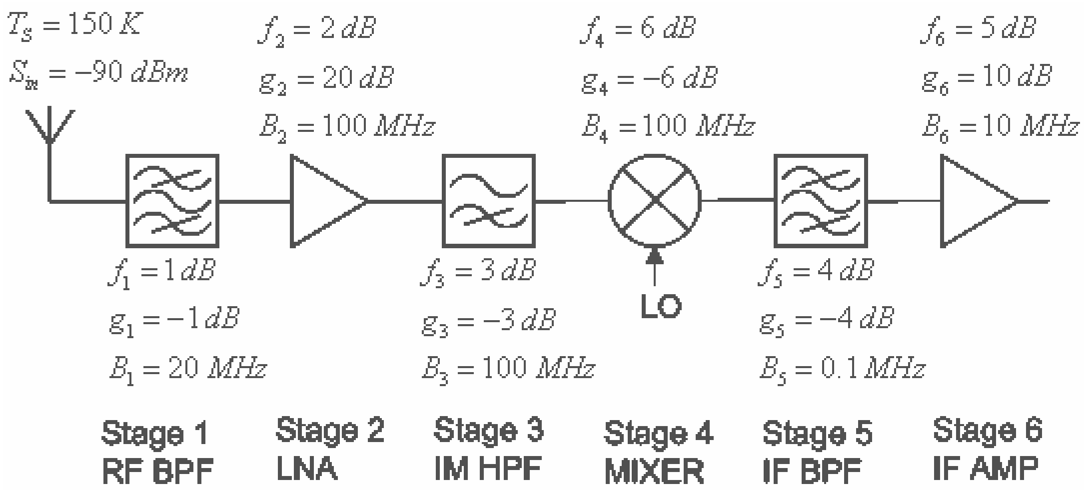
Chris Angove, July 2010

Thermal Noise Considerations of Cascaded Stages

This is an example of the front end of a typical communications receiver.



It comprises filters, amplifiers and a mixer. Each stage has a noise figure, gain and noise bandwidth: f_n, g_n and B_n respectively. The subscript n represents the stage number, starting at 1. Lower case symbols are used for logarithmic (dB) quantities, and upper case symbols for their linear equivalents. So for example

$$f_1 = 10\log_{10}(F_1) \quad dB$$
$$g_3 = 10\log_{10}(G_3) \quad dB$$

The linear form of noise figure, F_n is also known as noise factor and is often more convenient to use in the equations. The linear gain is strictly the linear gain magnitude but, by definition, we cannot express thermal noise in a mathematically predictable way with respect to time such as might be true with a continuous wave, so we do not use phase. Usually there will be some form of signal source, in this case supplied by an antenna which provides a signal input level to the receiver of $S_{in} = -90\ dBm$. The input noise temperature of the signal source here is 150 kelvin ($T_S = 150\ K$).

We should not be distracted by the fact that this example includes frequency conversion. This is simply because frequency conversion is very common in the front end of a super heterodyne receiver like this, where thermal noise is important and affects sensitivity. It does not change the principles of how sensitive front ends are affected by thermal noise. We assume that the signals under consideration are within the passbands of the filters and the normal operating frequency ranges of the mixer. However, the wideband nature of thermal noise means that it is also influenced by the rejection portions of the filter characteristics and by the rolloff portions of the amplifiers.

Assumptions

- This consideration is for thermal noise only, also known as additive white Gaussian noise (AWGN). We are not looking at the distortion effects caused by non-linearities such as harmonics and inter-modulation products.
- All stages are perfectly matched to each other.

- All stages are operating linearly.
- Stage noise factors are all specified for the IEEE standard noise input noise temperature, 290 K.

Noise Factor Definition

The noise factor F of a 2 port device, such as one of the cascaded stages shown above, is given by the linear ratio of the input signal to noise ratio S_i/N_i to the output signal to noise ratio S_o/N_o, so

$$F = \frac{S_i/N_i}{S_o/N_o} = \frac{N_o}{GN_i}$$

where G is the linear gain of the stage, so

$$G = \frac{S_o}{S_i}$$

The input and output noise powers are assumed to be integrated across identical (noise) bandwidths which are usually small fractions of the bandwidth of the device under test. Sometimes this bandwidth is called a *resolution* bandwidth. The noise at the input is assumed to come from a perfectly matched thermal noise source.

The following equation describing the thermal noise power coming from a perfectly matched thermal noise source is derived from Planck's theory of black body radiation with interpretations by Rayleigh and Jeans:

$$P_N = kTB$$

where

k is Boltzmann's constant, 1.38×10^{-23} joule per kelvin (J/K);

T is the absolute temperature (K);

B is the noise (resolution) bandwidth under consideration in hertz (Hz);

Again, it is important to remember that B is the chosen resolution bandwidth, not the bandwidth of any of the devices under consideration.

If N_i is the input noise power in linear units, we can substitute $N_i = kTB$ in to the noise factor definition above, so

$$F = \frac{S_i/N_i}{S_o/N_o} = \frac{N_o}{GN_i} = \frac{N_0}{GkTB}$$

Standard Noise Temperature

The equation above defining noise factor shows that the noise factor of a device depends on the input noise power and therefore of the noise temperature of the source connected to the input. So what noise power (or noise temperature) do we choose? In a practical system, to calculate the actual sensitivity performance we would use the actual noise power at the input. For the first stage of our cascade example it is 150 K. However, this might vary in other applications, so it is common for manufacturers to measure the noise factors of their products using an input noise temperature of 290 K. This is known as the standard input noise temperature, originally defined by the IRE, a predecessor of the IEEE. It can be produced by a matched impedance, very often 50 Ω, at about 17°C, not too far off room temperature and therefore easy to generate. In fact in warmer climates like California, 290 K is a reasonable approximation to a yearly average of ambient exterior air temperature. This accounts for its choice as many low noise amplifiers are externally mounted on antennas and much of the early theory was applied in California. The standard noise temperature is often represented by the symbol T_0, so

$$T_0 = 290 \quad K$$

The standard definition of noise factor replaces T with T_0, therefore

$$F = \frac{N_0}{GkT_0B}$$

Excess Noise Power

The excess noise power of a device is the equivalent noise power which, when placed at the output of the noisy device, is equivalent to the noise added by the device itself. The device would then be considered noise-free. Therefore the actual output noise power from a noisy device N_0 includes two components: that due to the amplified noise appearing at its input GkT_0B and that (excess) noise power added by the device itself, N_x. Therefore

$$N_0 = GkT_0B + N_x$$

By substituting for N_0 the noise factor defining equation becomes

$$F = \frac{N_0}{GkT_0B} = \frac{GkT_0B + N_x}{GkT_0B} = 1 + \frac{N_x}{GkT_0B}$$

Spectral Noise Density or Noise Power Density

Spectral noise density or noise power density (NPD) is a very useful way of considering the effects of thermal noise on cascades such as these. NPD is a measure of thermal noise power per unit bandwidth. From above, the NPD due to the excess noise power of a device with a standard noise temperature (T_0) source connected at its input is given by:

$$\frac{N_x}{B} = GkT_0(F-1) \quad W/Hz$$

This is referred to the output of the device.

In equations such as this where we have said nothing explicitly about units, the assumption is that we are using System International (SI) units. G and F (linear gain and noise factor respectively) are unit-less ratios. T_0 is expressed in Kelvin (K), and Boltzmann's constant k is expressed in joules per kelvin (J/K). Therefore in this case N_x/B will be a linear expression in watts per hertz (W/Hz). We will see later that NPDs are often converted or mixed with logarithmic units to come up with quantities such as (dBm/MHz). It does not matter which we use provided we are very careful to apply the correct conversions, we are very strict about stating the units used and we don't forget whether we are thinking in logarithmic or linear 'mode'.

Once we have the linear excess noise NPD at the output we can simply convert it if required to an equivalent NPD at the input of the device by dividing by the (linear) gain of the device G. Therefore the following expression is the excess noise NPD at the input of the same device:

$$\frac{N_x}{BG} = kT_0(F-1) \quad W/Hz$$

By referring the excess noise of the device either to the input or the output, the implication is that we have converted the device itself to one that is noise free. A noise-free device has a noise factor of unity or a noise figure of 0 dB. It will therefore, by definition, now have a noise factor of unity $F = 1$. It will still of course have the same gain as previously.

Noise Power in Terms of Temperature

From an earlier section we arrived at the well known equation for the noise power from a perfectly matched thermal noise source P_N:

$$P_N = kTB \quad W$$

This has been shown to be reliable at frequencies to well beyond 100 GHz. Furthermore it shows that there is a linear relationship between thermal noise power and bandwidth, for a constant noise temperature.

For the same source, the thermal NPD is therefore given by

$$NPD = \frac{P_N}{B} = kT \quad W/Hz$$

We know that Boltzmann's constant is a constant by definition. Therefore at a fixed noise temperature, the NPD of the thermal noise originating from the source is also constant. When we are referring to thermal noise and we know that it is from a correctly matched source, we only need to define its noise temperature. Note again, and this cannot be over-emphasized, B is the resolution bandwidth under consideration; it is not the bandwidth of any particular device under consideration.

Equivalent Input Noise Temperature

An alternative to expressing the noise performance of a device by an excess noise power density at its output or input is to define the equivalent input noise temperature T_e. It is generally understood that T_e may simply be referred to as the noise temperature of the device. From above, the noise factor of a device was defined as:

$$F = \frac{N_0}{GkT_0B} = \frac{GkT_0B + N_x}{GkT_0B} = 1 + \frac{N_x}{GkT_0B}$$

Remembering here that N_x is referred to the output and T_e to the input of the device, therefore

$$N_x = GkT_eB$$

So another way of expressing the noise factor is

$$F = \frac{N_0}{GkT_0B} = \frac{GkT_0B + N_x}{GkT_0B} = 1 + \frac{N_x}{GkT_0B} = 1 + \frac{GkT_eB}{GkT_0B} = 1 + \frac{T_e}{T_0}$$

We have therefore a simple way of converting between noise factor, based on the IEEE standard noise temperature T_0, and equivalent input noise temperature T_e:

$$F = 1 + \frac{T_e}{T_0}$$

Signal to Noise Ratio Through a Cascade

One of the advantages of using a superheterodyne architecture is to achieve good dynamic range. This requires good signal to noise ratio (SNR) performance. The following extract from an Excel spreadsheet shows an analysis of these stages for thermal noise, related parameters and the resulting effects on SNR. The calculations used in the spreadsheet use the theory that we have considered and will be described in the following sections. Note that some parameters are displayed to more decimal places than would normally be necessary to identify small but important differences.

Constants

At the top of the sheet are the constants with associated units: Boltzmann's constant, the standard noise temperature, and the noise temperature of the source.

The first column identifies the parameters being displayed again with units. Subsequent columns relate to stages 1 to 6 starting at the first stage as they were shown in the schematic diagram.

Boltzman Constant k =	1.38E-23	J/K
Standard Noise Temp. =	290	K
Source Noise Temp. =	150	K

	RF BPF 1	LNA 2	IMR HPF 3	MIXER 4	IF BPF 5	IF AMP 6
Stage Parameters						
Gain dB	-1.0	20.0	-3.0	-6.0	-4.0	10.0
Noise Figure dB	1.0	2.0	3.0	6.0	4.0	5.0
Noise Bandwidth MHz	20.0	100.0	100.0	100.0	0.1	10.0
Excess NPD at Output dBm/MHz	-120.8	-96.3	-117.0	-115.2	-116.2	-100.6
Excess NPD at Input dBm/MHz	-119.8	-116.3	-114.0	-109.2	-112.2	-110.6
Equivalent Input Noise Temp. K	75.1	169.6	288.6	864.5	438.4	627.1
Cascaded Parameters						
Gain dB	-1.0	19.0	16.0	10.0	6.0	16.0
Noise Figure (by Fris) dB	1.000	3.000	3.027	3.186	3.491	4.436
NPD Input dBm/MHz	-116.8	-116.1	-93.2	-96.1	-101.9	-105.5
NPD Output Due to Input Only dBm/MHz	-117.8	-96.1	-96.2	-102.1	-105.9	-95.5
NPD Output Due to Input + Excess dBm/MHz	-116.1	-93.2	-96.1	-101.9	-105.5	-94.4
Te Stage Only Reference Input K	75.1	213.5	3.6	21.7	43.8	157.5
Cumulative Te Reference Input K	75.1	288.6	292.3	314.0	357.8	515.3
Cumulative NF From Cumulative Te K	1.000	3.000	3.027	3.186	3.491	4.436
Effective Noise Bandwidth MHz	20.0	20.0	20.0	20.0	0.1	0.1
Signal Power						
Stage In dBm	-90.0	-91.0	-71.0	-74.0	-80.0	-84.0
Stage Out dBm	-91.0	-71.0	-74.0	-80.0	-84.0	-74.0
Noise Power						
Bandwidth Limited Output Noise Power dBm	-103.1	-80.2	-83.1	-88.9	-115.5	-104.4
SNR Output	12.1	9.2	9.1	8.9	31.5	30.4

Stage and Parameter Description Headings

The heading at the top of each column identifies which stage in the cascade the values in that column apply to. It starts with stage 1 (RF BPF) and finishes with stage 6 (IF AMP). The row headings on the left identify the parameters for the individual stages and for the cascade together with the units used.

Stage Parameters

The first 3 rows: gain, noise figure and noise bandwidth were entered directly from the data given for the stages in the schematic diagram. Typically this information would be obtained from the datasheets for the devices concerned.

Excess noise power density (NPD) at output was calculated using the following equation, with suitable processing to convert to units of dBm/MHz :

$$\frac{N_x}{B} = GkT_0(F-1) \quad W/Hz$$

For example, consider the LNA, stage 2. The logarithmic gain and noise figure are 20.0 dB and 2.0 dB respectively. Those correspond to a linear gain (G) of 100 and noise factor (F) of 1.585. k and T_0 are both known so substitution into the above equation gives $N_x/B = 2.341\times10^{-19} \quad W/Hz$. Converting to milliwatts per hertz (mW/Hz) this quantity was multiplied by 10^3 and to convert to mW per megahertz (mW/MHz) it was further multiplied by 10^6, equivalent to one multiplication by 10^9 or 2.341×10^{-10} mW/MHz. To convert that result to dBm/MHz the logarithm to base 10 is taken followed by multiplication by 10, giving the result $-96.3\, dBm/MHz$.

To obtain the NPD at the *input* of stage 2, the following equation may be used.

$$\frac{N_x}{BG} = kT_0(F-1) \quad W/Hz$$

This is equivalent to dividing the linear result for N_x/B by the linear gain of the stage. Alternatively subtracting the logarithmic gain (20 dB for stage 2) from the logarithmic NPD at the output (-96.3 dBm/MHz). The spreadsheet chose the latter option, arriving at the result -116.3 dBm/MHz.

To calculate the equivalent input noise temperature (or simply noise temperature) we need to start with the noise factor / noise temperature conversion equation as follows:

$$F = 1 + \frac{T_e}{T_0}$$

Making T_e the subject of this equation gives:

$$T_e = T_0(F-1)$$

Remembering that this equation uses the noise factor, not noise figure, we have already calculated the noise factor for stage 2 to be 1.585. Therefore substituting $F = 1.585$ and $T_0 = 290\, K$ gives $T_e = 169.6\, K$.

Cascaded Parameters

The rows headed 'Cascaded Parameters' relate to parameters at points along the cascade which are influenced by other stages in the cascade.

Cascaded Gain

The first cascaded parameter, gain, is simply the sum of the (logarithmic) gains of the stages prior to the point in the cascade that is under consideration. For example the cascaded gain at the output of stage 3 (IMR HPF) is: $-1.0 + 20.0 - 3.0\, dB = 16.0\, dB$. The same result may be obtained by calculating the product of the corresponding linear values, and converting the result to the logarithmic form.

Cascaded Noise Figure

The first row referencing cascaded noise figure uses Friis' equation directly which is, for linear values of noise factor and gain:

$$F_T = F_1 + \frac{F_2 - 1}{G_1}$$

Friis' equation is a common way of determining the equivalent noise factor of cascaded stages. F_1 and G_1 are the noise factor and linear gain respectively of stage 1, F_2 is the noise factor of stage 2. Initially, the equation may be applied to determine the equivalent noise factor of stages 1 and 2 combined. The spreadsheet displays noise figures not noise factors and logarithmic gains, not linear gains. Once the equivalent noise factor of stages 1 and 2 is calculated in this way, the result for F_T becomes the new stage 1 value (F_1) and stage 3 becomes stage 2 (F_2). The gain of stages 1 and 2 combined becomes the new G_1. The calculation then proceeds to the next stage. A similar calculation is applied repetitively for each of the stages in the cascade.

For example, considering stages 1 and 2 we have, in logarithmic units, $f_1 = 1\ dB$, $g_1 = -1\ dB$ and $f_2 = 2\ dB$. (We use lower case symbols for logarithmic units and upper case symbols for linear units.) Notice also that it does not matter that the first stage happens not to be an amplifier but an attenuator and therefore has a loss rather than a gain. Unless otherwise noted, the equations are defined in terms of gains, not losses, so a loss is simply expressed as a negative gain. To convert to linear (noise factor) values for stage 1, the linear to logarithmic conversion is $f_1 = 10\log_{10} F_1$ and the logarithmic to linear conversion is $F_1 = 10^{\frac{f_1}{10}}$. Similar equations apply to the other stages.

Similar definitions apply to the gain parameters, which are $g_1 = 10\log_{10} G_1$ and $G_1 = 10^{\frac{g_1}{10}}$. Applying the conversions to the example chosen, we have $F_1 = 1.26$, $G_1 = 0.79$ and $F_2 = 1.58$. By substituting these values into Friis' equation and then converting the result back to logarithmic units the final logarithmic result is $f_T = 3.0\ dB$ for the cascaded noise figure at the output of stage 2.

Cascaded NPD Input

In determining the input noise power density (NPD) of each stage, stage 1 must be treated differently from the others after which a common rule applies. The noise that stage 1 receives originates only from the source connected to the cascade, in this case set to a noise temperature of $150\ K$. We have seen that the thermal noise power P_N from a perfectly matched noise source at noise temperature T_S is given by

$$P_N = kT_S B \quad W$$

Therefore the NPD of the same source for $T_S = 150\ K$ is given by

$$\frac{P_N}{B} = kT_S = 2.07\times 10^{-21}\ W/Hz$$

The units in this case are watts/hertz (W/Hz) because we have not applied any conversion factor to the values for k and T_S which were expressed in SI units. In the spreadsheet, we have chosen to display the same quantity in dBm/MHz. The conversion is applied in 3 stages:

- watts/hertz (W/Hz) to watts/megahertz (W/MHz), multiply by 10^6.
- W/MHz to milliwatts/megahertz (mW/MHz); multiply by 10^3.
- mW/MHz to dBm/megahertz (dBm/MHz); take the logarithm base 10 of the result and multiply by 10.

By applying this conversion to the input noise temperature of $T_S = 150\ K$, yields a result for the NPD of $-116.8 \quad dBm/MHz$. For later stages, the cascaded NPD at the input is simply the same as the cascaded NPD at the output of the previous stage. Output NPDs are discussed in the next section.

NPD at Output

The NPD at the output of a noisy device such as an amplifier, will comprise the (linear) sum of two NPD components:

- that due to the simple amplification of the NPD alone that was applied to the input.
- that added by the amplifier itself, also known as the excess NPD.

Importantly, these components are uncorrelated. That means that there is no phase relationship between them or they originate from completely independent sources. Using logarithmic units, the first component is simply the input NPD plus the gain of the device concerned, remembering again that if the device actually has a loss, in logarithmic terms it is treated as a negative gain. So, for example, a 10 dB attenuator is equivalent to an 'amplifier' with a gain of – 10 dB. This can be seen from the first column of the spreadsheet, applicable to stage 1 in which, as we have seen, the input NPD is $-116.8 \quad dBm/MHz$ and the output NPD (due to the input NPD only) is $-117.8 \quad dBm/MHz$. Stage 1 has a loss of 1 dB, equivalent to a gain of – 1 dB.

The NPD added by the inherent thermal noise of the amplifier itself, or the excess NPD, has already been calculated and is shown in one of the rows under the stage parameters. For example, for stages 2 and 3 it is $-96.3 \quad dBm/MHz$ and $-117.0 \quad dBm/MHz$ respectively. In each case it was referred to the output of the stage concerned.

NPD at Output Due to the Input NPD and Excess NPD

As the two output NPD components are uncorrelated, we do not have to worry about any phase relationship between them because this is entirely random. The resulting noise power is the linear power sum of the two components. Therefore the values that are in logarithmic form must be converted to linear, added and converted back to their logarithmic form.

Take for example, the total output NPD from stage 2 is $-93.2 \quad dBm/MHz$. This is the linear power sum of the component due to the input NPD only ($-96.1 \quad dBm/MHz$) and of the output excess noise NPD of the stage stand-alone ($-96.3 \quad dBm/MHz$). Each value may be converted to linear NPD units such as milliwatts/megahertz (mW/MHz) by dividing by 10, then raising 10 to that power. The results therefore become $10^{\frac{-96.1}{10}} = 2.4547 \times 10^{-10} \quad mW/MHz$ and $10^{\frac{-96.3}{10}} = 2.3442 \times 10^{-10} \quad mW/MHz$ respectively. The linear sum of these two quantities is $4.7989 \times 10^{-10} \quad mW/MHz$. Converting this to logarithmic units yields the result $-93.2 \quad dBm/MHz$ which agrees with the result shown in the table.

Te Stage Only Referenced to Input

None of the stages in this example is noise-free since no noise figure is 0 dB. Each may therefore be represented by a finite equivalent input noise temperature (T_e) at the same time replacing the stage by a theoretical noise free stage with the same gain. T_e is given by

$$T_e = T_0(F-1)$$

For example, for stage 1 only which has a noise figure of 1 dB, we can add the suffix '1' to the subscripts so that

$$T_{e1} = T_0(F_1-1)$$

By substitution and changing from noise figure to noise factor at the same time gives the following result which is shown in the spreadsheet:

$$T_{e1} = T_0\left(F_1 - 1\right) = 290\left(10^{\frac{1}{10}} - 1\right) = 75.1 \quad K$$

The input to stage 1 is also the input for the cascade as a whole so no further modification is necessary for the cascaded result. However, for a similar consideration of stage 2 whose noise figure is 2.0 dB, we have

$$T_{e2} = T_0\left(F_1 - 1\right) = 290\left(10^{\frac{2}{10}} - 1\right) = 169.6 \quad K$$

T_{e2} by definition is located at the input of the stage it relates to and therefore in this case at the output of stage 1. To refer it to the input of the *cascade* as a whole therefore it must be *divided by the linear gain of stage 1.* The logarithmic *gain* of stage 1 is – 1.0 dB, so the linear gain is given by

$$G_1 = 10^{\frac{-1}{10}} = 0.794$$

T_{e2} referred to the input of the cascade is therefore

$$\frac{T_{e2}}{G_1} = \frac{169.6}{0.794} = 213.5 \quad K$$

A similar procedure is applied to later stages, in each case noting that the noise temperature must be divided by the combined linear gain of all of the stages before it in order to refer it back to the input of the whole cascade. This has been performed in the ‘Cascaded Parameters’ part of the table for each of the stages individually in the row headed ‘Te Stage Only Ref. Input K’.

Cumulative Te Referenced to Input K

We saw in the previous section how the equivalent input noise temperature of each stage may (individually) be referred back to the input of the cascade. One very useful property of using noise temperatures as opposed to noise factors is that they are directly proportional to *linear* noise power and may therefore be added to generate an equivalent noise source if two or more such values are referred to the same point. In the row which is headed ‘Cumulative Te Reference Input K’ the referred noise temperatures have been accumulated in this way by linear addition so that each column provides the result of this addition for the part of the cascade up to that point. For example, the noise temperature of the first stage referred to the input is 75.1 K and that of the second stage is 213.5 K. The noise temperature of both the first and second stages combined is therefore the sum of these individual contributions or 288.6 K as shown in the spreadsheet. To obtain the equivalent noise temperature of the first 3 stages, the referred noise temperature of the third stage (3.6 K) must be similarly added, giving 292.3 K. A similar procedure is applied for the remaining stages.

Cumulative NF from Cumulative Te dB

We have seen that there is a simple relationship between noise factor, based on the standard input noise temperature, and equivalent input noise temperature as follows

$$F = 1 + \frac{T_e}{T_0}$$

Each value which is in the row headed ‘Cumulative NF from cumulative Te dB’ simply applies this equation to the individual values of the row above, then converting from the linear noise factor to the logarithmic noise figure, expressed in dB. For example, consider again the equivalent input noise temperature for the first 3 stages, a value of 292.3 K. Using the above equation to calculate the equivalent noise factor for the same 3 stages combined, we have

$$F_{13} = 1 + \frac{T_e}{T_0} = 1 + \frac{292.3}{290} = 2.0079$$

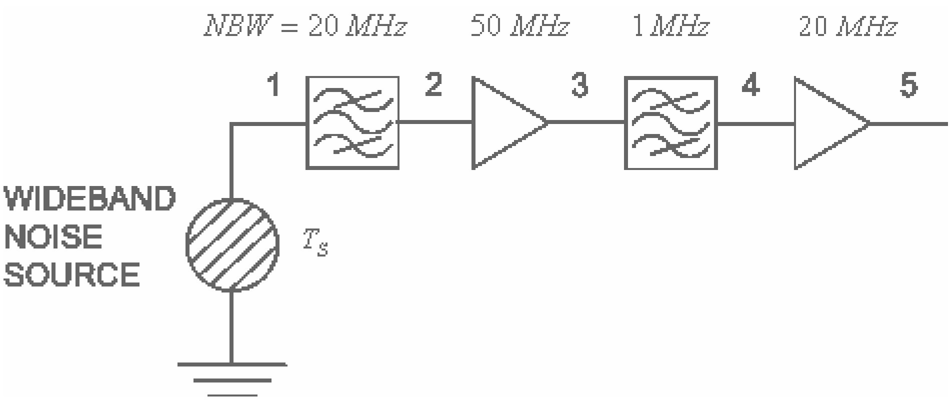
Converting the result to dB

$$f_{13} = 10\log_{10}(2.0079) = 3.027 \quad dB$$

Effective Noise Bandwidth MHz

In a typical superheterodyne architecture like that we are considering, filters would be used to reduce sources of noise, including thermal noise. In our cascade there are 3 filters in total, one at each of the stages 1, 3 and 5. The first at stage 1 is a relatively crude, wideband but effective 'roofing' bandpass filter, the stage 3 filter is an image rejection filter and the third at stage 5 is an IF bandpass filter, sometimes called a channel filter. Typically the noise bandwidth of each of these filters will be somewhere near their – 3 dB bandwidth. The other stages, although designed to be reasonably wideband, will have some practical limit to their bandwidth and therefore each will have its own effective noise bandwidth. The effective noise bandwidth of each of the stages in megahertz is shown in the spreadsheet Stage Parameters section. For the purposes of thermal noise analysis we need not worry about the differing *absolute* frequencies at different points along the cascade, a property of the superheterodyne receiver, by definition. The effective noise bandwidths are all considered to be normalized to the center frequency of either the RF (before the mixer) or the IF (after the mixer) as necessary for their respective sections. Because, for a fixed noise temperature, thermal noise power density is effectively constant up to in excess of 100 GHz, it is only necessary to consider noise bandwidths and not the absolute frequency of the noise.

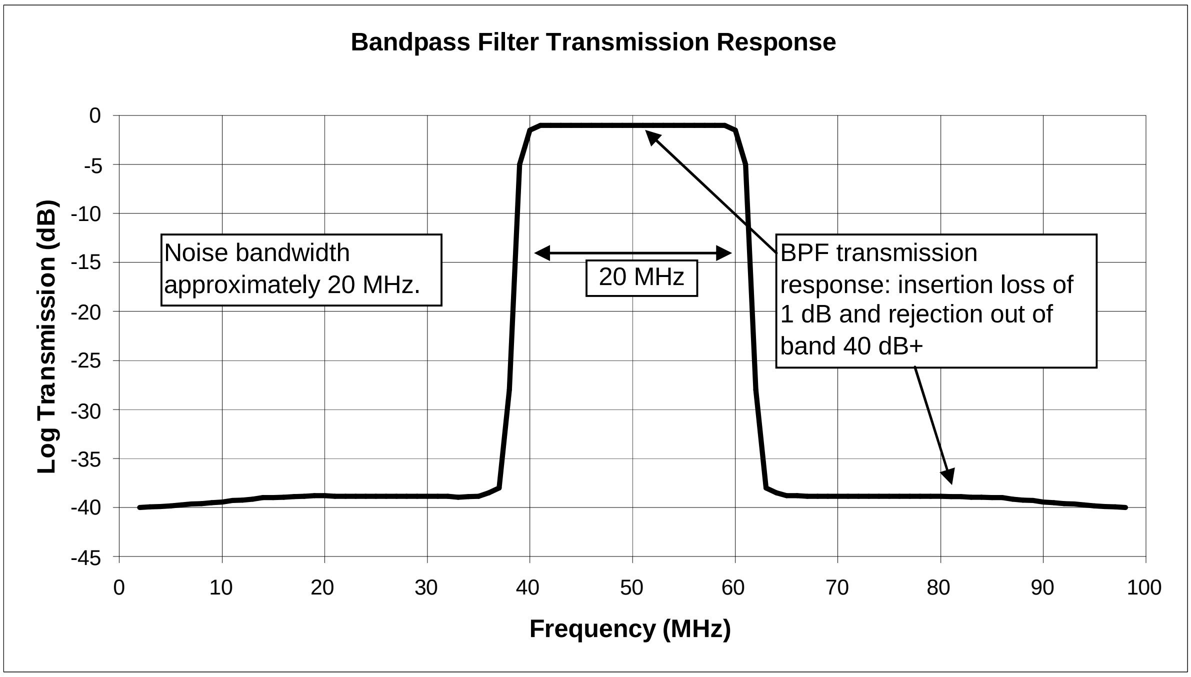
Furthermore, and to make life easier we are going to assume that the noise bandwidths of the filters get progressively smaller as the signal (and noise) passes further along the cascade. Ideally, immediately prior to the point at which the signal is detected would be a filter with the narrowest noise bandwidth. Usually in superheterodyne architectures filters can be readily designed to achieve this but the noise bandwidths of the stages in between are less predictable and usually much wider. Let us digress here and look at this in more detail with a simplified example, shown schematically below. This is a theoretical cascade without frequency conversion and we assume that the frequency response of each stage has the same center frequency. In fact the noise source and first stage are identical to those we considered in the first cascade. A wideband noise source at a noise temperature $T_S \quad K$ is applied to the input of the first stage.



The noise bandwidths of stages 1 through to 4 are: 20 MHz, 50 MHz, 1 MHz and 20 MHz respectively. Each stage has a finite noise figure so therefore makes an individual contribution to the accumulation of noise passing through the cascade.

The logarithmic transmission response against frequency for a good quality bandpass filter such as that used for stage 1, is shown in the following diagram, in this case for a – 3 dB bandwidth of approximately 20 MHz.

Typically its noise bandwidth would be of the same order. A characteristic such as this may be measured using a swept continuous wave (CW) applied to the input and a power measuring instrument at the output. In terms of noise analysis this would be equivalent to applying a know NPD at the input and measuring the response at the output with a suitable measuring instrument, such as a spectrum analyzer. We have decided to use logarithmic units of transmission quite simply because they can represent large dynamic ranges more easily and the arithmetic is easier. The in-band insertion loss is 1 dB and the out of band rejection is approximately 40 dB.



Supposing the noise temperature of the noise source at the input is at 150 K. There would then be two sources contributing to the noise measured at the output of the filter:

- that due to the noise source at a noise temperature of 150K;
- the excess noise of the filter itself on account of its finite insertion loss of 1 dB.

The NPD due to the source considered alone is given by the following equation in linear units of W/Hz

$$\frac{P_N}{B} = kT_S = 2.07 \times 10^{-21} \; W/Hz$$

Let us convert this to logarithmic units of dBm/MHz. The linear result must be multiplied by 10^3 to obtain units of mW/Hz, then by 10^6 to get mW/MHz, its logarithm to base 10 taken and the result multiplied by 10. The result, say p_N/B_{RES} is given by:

$$\frac{p_N}{B_{RES}} = 10\log_{10}\left(kT_S \times 10^3 \times 10^6\right) = -116.8 \quad dBm/MHz$$

At the input to the filter, this is shown in the following log transmission against frequency graph with the same frequency scaling that was used for the filter transmission response. Notice however that the vertical scale for NPD is now in an *absolute* logarithmic unit of dBm/MHz.

The bandpass filter has a finite noise figure of 1 dB. Therefore it must, from the definition of noise figure, contribute noise to the cascade and any signal that might be present. We have seen that the following equation provides the NPD in the form of excess noise power at the output from a noise device of noise factor $F = 10^{\frac{1}{10}} = 1.259$ and linear gain $G = 10^{\frac{-1}{10}} = 0.794$.

$$\frac{N_x}{B} = GkT_0(F-1) \quad W/Hz$$

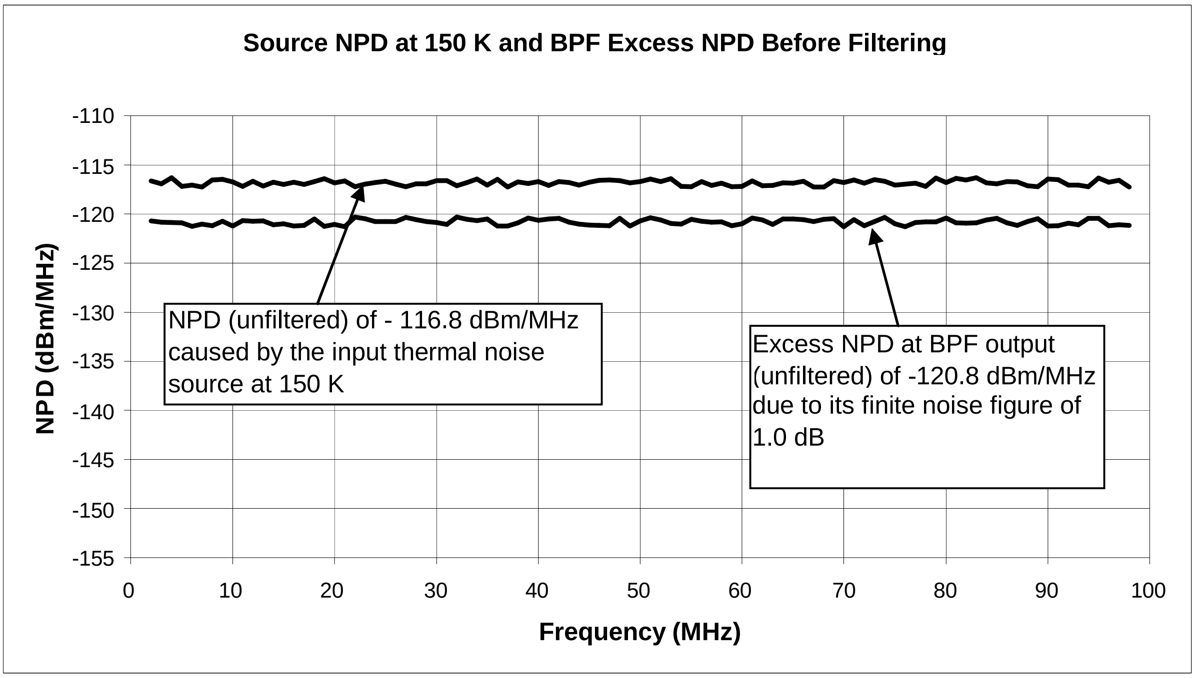
By performing the calculation for this filter, in linear units, we have

$$\frac{N_x}{B} = 0.794 \times 1.38 \times 10^{-23} \times 290(1.259-1) = 8.23 \times 10^{-22} \quad W/Hz$$

Converting to the same logarithmic units as previously.

$$\frac{p_N}{B_{RES}} = 10\log_{10}\left(8.23 \times 10^{-22} \times 10^{3} \times 10^{6}\right) = -120.8 \quad dBm/MHz$$

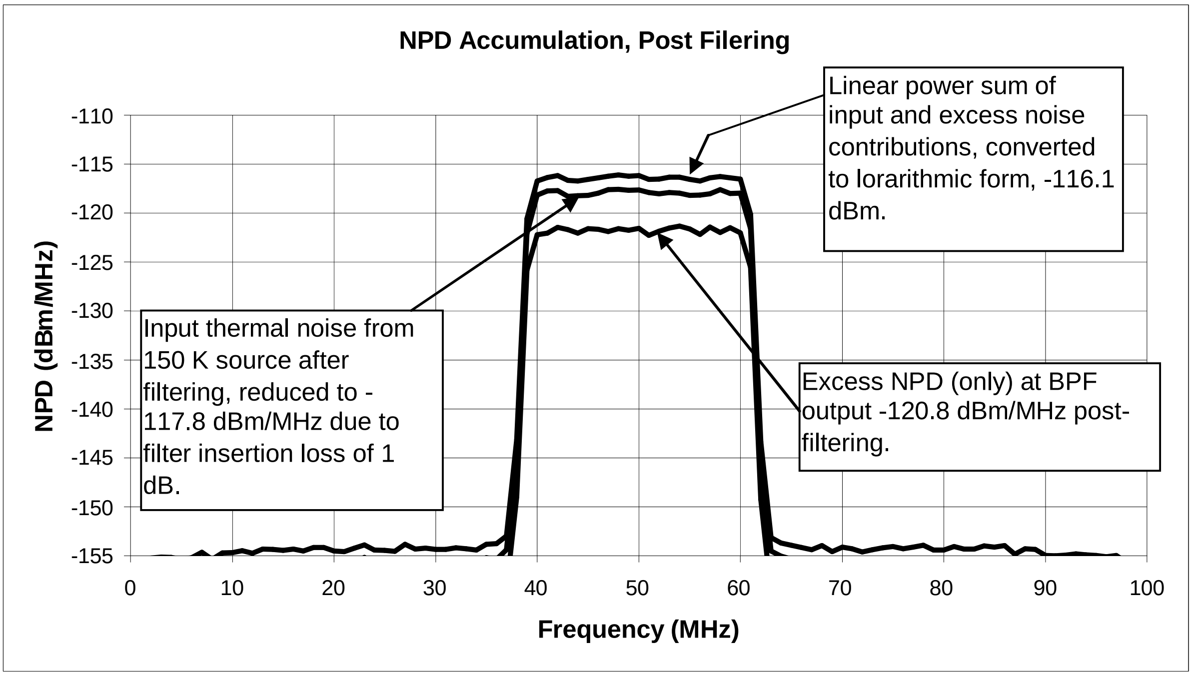
This level is also shown on the graph.



At the output of the filter within its passband, the input noise NPD is attenuated by 1 dB due to the (in-band) insertion loss of the filter, giving $-117.8 \quad dBm/MHz$. The excess noise NPD of -120.8 dBm/MHz does not require adjustment for insertion loss as it is already referred to the output. The total noise power is the *linear* sum of the two NPD contributions just calculated, $-117.8 \quad dBm/MHz$ and $-120.8 \quad dBm/MHz$ performed in the following way

$$\frac{p_{NT}}{B_{RES}} = 10\log_{10}\left(10^{\frac{-117.8}{10}} + 10^{\frac{-120.8}{10}}\right) = -116.1 \quad dBm/MHz$$

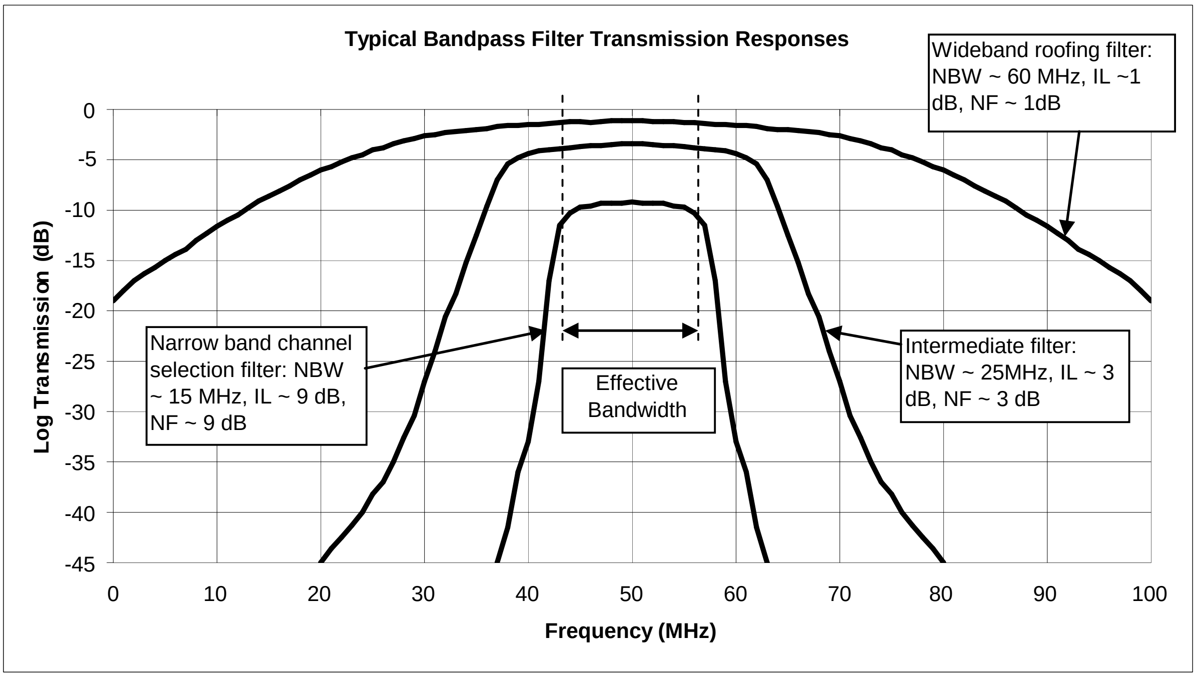
The figure below shows the NPD against frequency characteristic at the output of the bandpass filter which is also shown . The total noise NPD of -116.1 dBm/MHz appears within the passband. This agrees with the spreadsheet value.



The same principles are applied to determine the NPD at later stages in the cascade.

The second stage is a noisy amplifier with a noise bandwidth of 50 MHz. Although its noise bandwidth is much wider than the passband of the filter in front of it, we are using noise power densities rather than absolute noise powers. That means that we can look later in the cascade for narrower filters: and there is one of noise bandwidth 1 MHz, after the amplifier. Remembering that we have assumed that all devices in the cascade have the same center frequency, the response of the 1 MHz filter would fall in the middle of the passband of the 20 MHz filter. Therefore if we only consider the middle 1 MHz of each of the devices in the cascade we can apply exactly the same techniques that we did for the 20 MHz filter. It does not even matter what units we chose for the NPD. For example, we could use dBm/MHz if the final filter had bandwidths of 50 MHz, 10 MHz or even 0.234 kHz, it would just be a little more difficult to calculate the total noise power in the latter case as we will see later.

The following figure shows the transmission responses of three bandpass filters of reducing bandwidth, designed using the same technology with the frequency scaling normalized to the same center frequency. Notice that the insertion loss of each filter tends to increase as the bandwidth decreases. This is typical for such filters and is a result of the increased number of sections necessary for the narrower bandwidths. These filters are typical of what might be found in a superheterodyne front end such as we have been considering. Provided that the signal passes through them in order of reducing bandwidth, the effective bandwidth is that of the narrowest filter, shown by the dotted lines.



This analysis shows how the ultimate bandwidth achieved is determined by the last and narrowest filter that is encountered. Why then do we not simply put in a very narrow band filter right at the front of the cascade? There are several reasons:

We have not shown any frequency conversion. Normally the front end of the cascade would be operating at a much higher frequency than the intermediate frequency (IF). For the same technology, the complexity of a filter is approximately inversely proportional to its percentage bandwidth. So if the front end frequency was 1 GHz, a 10% bandwidth filter would have a passband of 100 MHz. The same complexity of filter at an IF of 100 MHz would have a bandwidth of 10 MHz. To provide a 1% filter bandwidth of 10 MHz at 1 GHz would be quite a challenge and would probably need several stages of very high Q factor resonators and present a significant loss in the passband. Such a high loss at the front end of a cascade could degrade the overall noise figure substantially. Also, if the front end filter is too narrow it will restrict the tuning ability of the superheterodyne architecture. Larger percentage bandwidth filters are easier to design, cheaper and generally have a smaller insertion loss that smaller percentage bandwidth equivalents.

Adding a Signal: Signal to Noise Ratio (SNR)

Now we are ready to add a signal to the input of the cascade in order that we may calculate the signal to noise ratios (SNRs) at various points along the cascade. Although signals in practice might carry modulation and therefore occupy a finite spectral bandwidth, it is quite normal in cascade analysis to assume the signal is a perfect continous wave (CW) and theoretically occupy zero bandwidth. However, the ultimate (narrowest) bandwidth filter determines both the absolute noise power contribution (that part which contributes to the SNR) and the bandwidth available to carry modulation. Again we have a tradeoff. If the bandwidth of this filter is too narrow we have good SNR performance but limited channel bandwidth (or capacity). It it is too wide the reverse is true.

Returning to the original cascade the input signal level is at – 90 dBm. Using logarithmic units, the first stage attenuates it by 1 dB giving – 91 dBm. The second stage amplifies it by 20 dB resulting in – 71 dBm at the output of the second stage. The signal is modified similarly through the remaining stages.

Effective Noise Power through the Cascade

We have just defined how the signal levels are calculated as it passes through the stages. All we need now are the absolute noise powers in order to calculate the SNR values through the cascade.

We have already calculated the (total) NPD values through the cascade and included them in the spreadsheet. To obtain the absolute noise power at a particular stage, assuming linear values, the NPD must be multiplied by the effective noise bandwidth. At points passing through the cascade, we have defined the effective noise bandwidth at the point concerned to be either the same as the noise bandwidth of the stage or the effective noise bandwidth of the previous stage, whichever is the smaller. This is a reliable assumption provided the noise bandwidths of the bandpass filters become successively smaller whilst progressing through the cascade, which is the normal architecture. Equivalent noise bandwidths calculated in this way are shown in the spreadsheet.

The spreadsheet shows absolute noise powers which have been calculated in this way. For example, at the output of stage 2 the effective noise bandwidth is 20 MHz. At the same point the total NPD is, in logarithmic units, - 93.2 dBm/MHz. Therefore in a bandwidth of 1 MHz the noise power is – 93.2 dBm. In 20 MHz the linear power is 20 times higher or, in logarithmic units $20\log_{10}(20) = 13.01 \quad dB$. The result is therefore $-93.2 + 13.01 = -80.2 \quad dBm$ and shown in the spreadsheet. The SNR values passing through the cascade is the (linear) ratio of signal power to noise power at the same point. SNR may be expressed in logarithmic (dB) units instead by subtracting the noise power from the signal power. The bottom row of the spreadsheet shows the logarithmic SNR values calculated in this way. For example, at the output of stage 3 the signal power is – 74.0 dBm and the noise power is – 83.1 dBm. The logarithmic SNR is therefore given by $-74.0 - (-83.1) = 9.1 \quad dBm$. Notice how the SNR can actually be increased with appropriate choice of filter. For example, the SNR at the input to the narrow band filter, or the output of stage is 4, is 8.9 dB but increases to 31.5 dB at its output, stage 5. Although this filter has a significant noise figure of 4 dB, the extra noise resulting from this is more than offset by the reduced noise power contributed to the SNR by virtue of its narrow bandwidth.

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