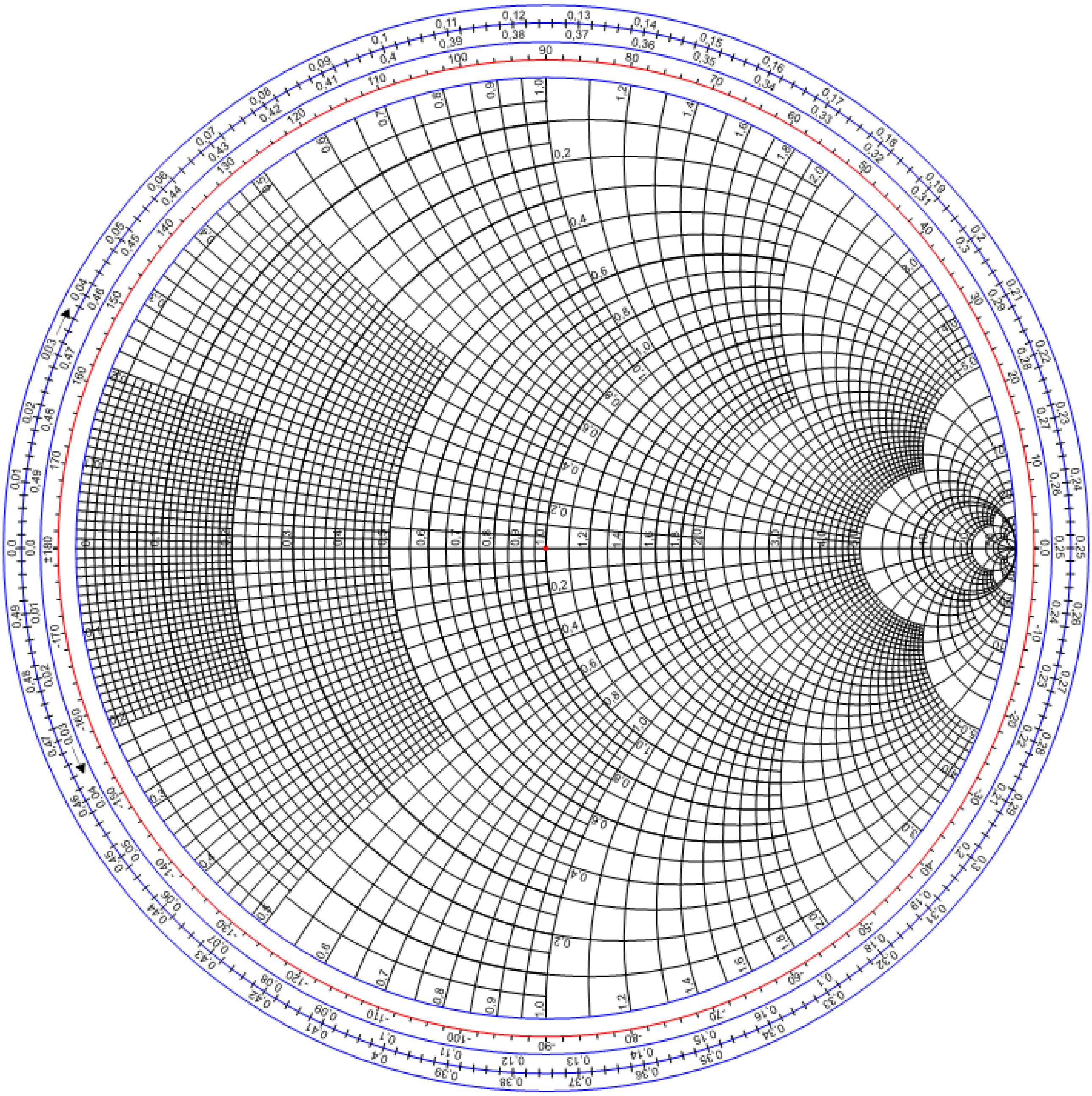


The Smith Chart

CONTENTS

TABLES

FIGURES

1 Introduction

1.1 International System of Units

This article uses the International System of Metric Units (SI Units) which are listed in Table 1-1 together with parameter names, commonly used symbols, SI units and SI base units [26] [1]. The other parameters and symbols used are defined within the text.

Table 1-1 Common electrical parameters, symbols, SI derived units and base units.

Parameter		SI Derived Unit(s)		SI Base Unit(s)
Name	**Symbol**	**Name**	**Symbol**	
Capacitance	C	farad	F	$kg^{-1}\,m^{-2}\,s^4\,A^2$
Inductance	L	henry	H	$kg\quad m^2\,s^{-2}\,A^{-2}$
Voltage	V	volt	V	$kg\quad m^2\,s^{-3}\,A^{-1}$
Electric Current	I	ampere	I	A
Permittivity	ϵ	farad/metre	F/m	$kg^{-1}\,m^{-3}\,s^4\,A^2$
Permeability	μ	henry/metre	H/m	$kg\quad m\quad s^{-2}\,A^{-2}$
Time	t	second	s	s
Mass	m	kilogram	kg	kg
Length	l	metre	m	m
Frequency	f	hertz	Hz	s^{-1}
Angular Frequency	ω	radian/second	rad/s	s^{-1}
Q-factor	Q	(unitless)	$(unitless)$	$(unitless)$
Power	P	watt	W	$kg\quad m^2\,s^{-3}$
Energy	U	joule	J	$kg\quad m^2\,s^{-2}$
Impedance	Z	ohm	Ω	$kg\quad m^2\,s^{-3}\,A^{-2}$
Resistance	R	ohm	Ω	$kg\quad m^2\,s^{-3}\,A^{-2}$
Reactance	X	ohm	Ω	$kg\quad m^2\,s^{-3}\,A^{-2}$
Admittance	Y	siemen	S	$kg^{-1}\,m^{-2}\,s^3\,A^2$
Conductance	G	siemen	S	$kg^{-1}\,m^{-2}\,s^3\,A^2$
Susceptance	B	siemen	S	$kg^{-1}\,m^{-2}\,s^3\,A^2$

A list of multiplication factor symbols which may be used immediately before the SI unit symbols in Table 1-1 is shown in Table 1-2 [26].

Table 1-2 SI unit multiplication factors

Prefix	Symbol	Factor	Power of 10
tera	T	1 000 000 000 000	10^{12}
giga	G	1 000 000 000	10^9
mega	M	1 000 000	10^6
kilo	k	1 000	10^3
hector	h	100	10^2
deca	da	10	10^1
(none)	(none)	1	10^0
deci	d	0.1	10^{-1}
centi	c	0.01	10^{-2}
milli	m	0.001	10^{-3}
micro	μ	0.000 001	10^{-6}

[1] An opportunity to test your memory of physics equations and hopefully agree with the base units.

Prefix	Symbol	Factor	Power of 10
nano	n	0.000 000 001	10^{-9}
pico	p	0.000 000 000 001	10^{-12}

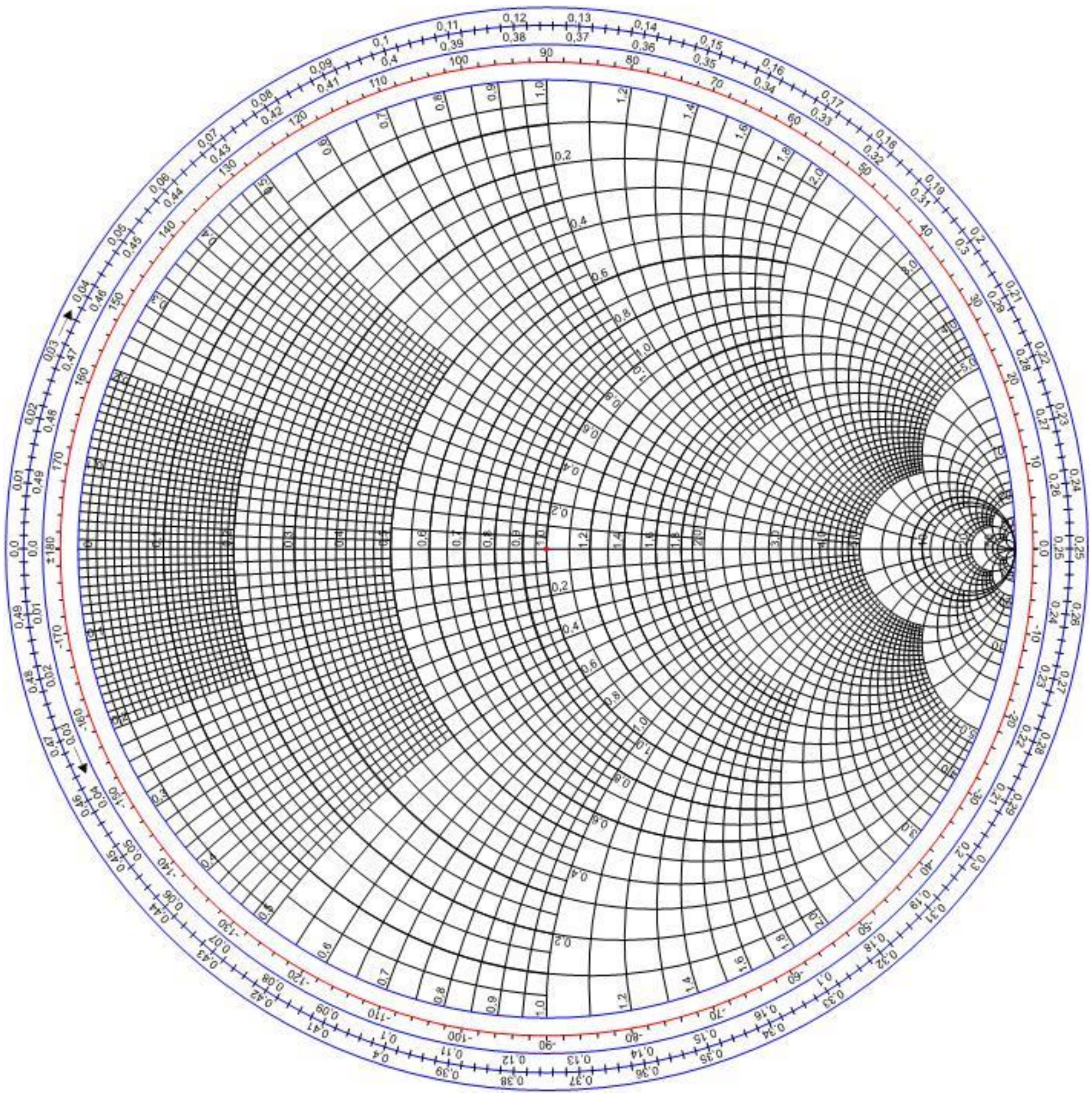


Figure 1-1 A blank example of the Smith Chart, invented by Philip H. Smith in 1936, showing the reflection coefficient unity radius region. This is the most common orientation, typically used for normalized impedance.

2 Overview

The Smith Chart, an example of which is shown in Figure 1-1, was invented by Phillip H. Smith in 1936. This is a graphical aid used to help solve radio frequency (RF) and microwave engineering problems involving imperfectly matched sources and loads [1][2]. The copyright to the Smith Chart is now held by the Microwave Theory and Technology Society (MTT-S), an affiliate of The Institute of Electrical and Electronics (IEEE) [2]. We acknowledge its use here with thanks, free of charge, for educational purposes.

Figure 1-1 shows an example of a blank unity radius Smith Chart. The scaling displayed may be used for normalized impedance (z) or normalized admittance (y) to represent values of the complex reflection coefficient (ρ) one or more frequencies. Finite values of ρ occur for any connection between a source and an

[2] All (real) loads are imperfectly matched. Hopefully many of them we design will be reasonably matched.

imperfectly matched load[3]. These may be mapped direct on to the Smith Chart format, Figure 1-1, or an equivalent linear polar diagram[4]. Any point on a z Smith Chart may be translated to the equivalent on a y Smith Chart, by rotation at constant radius through 180 degrees about the center of the Smith Chart ($|\rho| = 0$, $z = 1 \pm j0$ or $y = 1 \pm j0$). Then the scaling is changed to normalized admittance until a possible further translation is required from y back to z [5]. Alternatively, the Smith Chart may be scaled to simultaneously represent both z and y, typically with different coloured prints, though this can cause cluttering.

The outer circumferential scaling represents distances in electrical wavelengths along a loss-free transmission line towards (clockwise) or away from the load. The characteristic impedance of the transmission line must be the same as used on the Smith Chart. This applies if the component dimensions (including cables and connectors) are significant fractions of the shortest wavelength under consideration, also known as *distributed component conditions*. If the component dimensions are small fractions of the shortest wavelength, distributed component effects become negligible and the electrical behaviour may be described as *lumped element conditions*.

The popularity of the Smith Chart has grown steadily over the years and it is still widely used today, not only as an RF design aid, but to graphically demonstrate the variations of RF parameters over frequency. These include normalised impedance, normalised admittance, complex reflection coefficients, scattering parameters of the form S_{nn}, noise figure circles, constant gain contours and regions at risk of instability [7] [8] [14]. The Smith Chart is most frequently used within the unity radius region as shown in Figure 1-1, in which the magnitude of the reflection coefficient is between 0 ($|\rho| = 0$) and 1 ($|\rho| = 1$). The 'expanded' Smith Chart magnifies the region close to the center and the 'condensed' Smith Chart includes the region for $|\rho| \succ 1$. The expanded Smith Chart provides improved accuracy in dealing with networks of imperfect but still good match. The condensed Smith Chart includes the 'negative resistance' region of potential instability if an active device is present [14].

For all Smith Chart work a suitable system impedance, or characteristic impedance, Z_0, must be chosen to provide a reference for normalization. By definition, this is assumed to be purely resistive. Despite this, it is common practice to still refer to it as an impedance. The most common characteristic impedance used in today's electronics systems is probably 50 Ω. Other common examples are 75 Ω, 100 Ω, 300 Ω and 600 Ω. Many 50 Ω and 75 Ω systems happen to be coaxial (unbalanced) and many 300 Ω and 600 Ω systems happen to be balanced, but Smith Charts and transmission line theory apply equally to either. Alternatively, a system admittance (Y_0) may be chosen, although this is less common. This has the reciprocal relationship to system impedance:

$$Y_0 = \frac{1}{Z_0} \qquad (2.1)$$

For the best accuracy, it is good practice to use a characteristic impedance close to the nominal impedance of the networks and devices under development. For example, bipolar RF power transistors may have input and output impedance magnitudes in the order of just a few ohms so 5 Ω may be a better choice than 50 Ω.

Problems using the Smith Chart can only be solved one frequency at a time. This may be adequate for narrow band applications, but for wider bandwidths it is usually necessary to use the Smith Chart at other frequencies

[3] Note the use of lower case for normalized values, z and y. For describing Smith Charts, often the adjective 'normalized' is omitted or the upper case symbols (Z or Y) are used instead.
[4] The ρ scaling is not explicitly displayed as this would excessively clutter the diagram.
[5] The $y = 1 \pm jb$ conjugate matching circle (not shown) passes through 1 and 0 instead of 1 and infinity.

across the operating frequency band to form a locus of points. This provides the following visual information of the imperfect load across frequency:

- the reactive and resistive behaviour;
- the degree of mismatch from the characteristic impedance.

We noted that the best Smith Chart accuracy is achieved for component impedances close to the characteristic impedance, so that their normalised impedances are close to unity $(1 \pm j0)$. Therefore, if we were working with devices (cables, connectors, amplifiers, attenuators, filters etc.) *nominally designed* for 50 Ω, a good choice of Z_0 would also be 50 Ω. 'Nominally designed' means that the devices were designed with interface (input and output) impedances as close as possible to Z_0. In practice, however, there will be compromises and their actual impedances will never be exactly Z_0, especially after accounting for factors including frequency, temperature, DC voltages and bias conditions.

Today, the Smith Chart is less frequently used in its manuscript, pencil and paper form, but it is a standard output format option included for many computer aided design (CAD) RF and microwave applications and test equipment, especially the vector network analyzer (VNA).

The construction and use of the Smith Chart will be described in more detail in Chapter 4, once we have paused to study the equations associated with imperfectly matched transmission lines, generically referred to as the transmission line equations (TLEs)[6].

3 The Transmission Line Equations

The transmission line equations (TLEs) were originally developed to understand how telegraph signals degraded through transmission over very long cables, formed from conductors, even thousands of miles. Today, we will rarely find examples of telegraphy outside of a museum, but widespread deployment of data transmission infrastructure means the theory and applications of transmission lines have never been more important [9] [11] [12] [21][7].

The TLEs apply to transmission lines constructed from good electrical conductors, typically copper. There are always two conductors: one for the forward current and the other for the return current[8]. They do not include wave mode transmission lines like waveguides and fiber optic cables[9].

The circuit concept of 'maximum power transfer', when the source impedance is identical to the load impedance, is too elementary in practice when we move to real RF (AC) circuits and higher frequencies. A perfect 'match' of this type cannot be achieved in practice across a reasonable bandwidth so we must accommodate imperfect matches and develop various methods to improve them.

3.1 Electrical Dimensions

Electrical dimensions simply mean the dimensions expressed in wavelengths, using the shortest operating wavelength (the highest frequency). In radio frequency (RF) engineering, simplifications can often be made after considering the *electrical* dimensions of the components we plan using.

---

[6] Good familiarity of the transmission line equations, not necessarily the proofs, is strongly recommended before using Smith Charts.
[7] Data transmission cables are transmission lines with very challenging broadband requirements.
[8] Early submarine cables used ground (earth) return for the signal currents. More modern submarine cables still use ground return but for the power feed to the line amplifiers (repeaters).
[9] There are similarities to conductor transmission lines but we need to return to Maxwell's equations so we will not be doing that today.

The Smith Chart constructions are simplified if we can reliably neglect the spatial phase. This is scaled in the form of wavelengths, which is a function of spatial phase on the circumferential scales. If the shortest wavelength is *much greater than* the dimensions of the components, including connectors and interconnecting cables, we may not even need to consider the spatial phase component. We will be using *lumped element conditions*.

To answer the question, “How much is ‘much greater than’?”, the widely accepted rule of thumb is ten times or greater. In most well designed RF circuits however this factor will be comfortably greater than ten times. If we cannot neglect spatial phase we are said to be using *distributed element conditions*.

Assuming lumped element conditions, Figure 3-1 shows circuit schematics for connecting an imperfectly matched source (Z_0) on the left to a load (Z) on the right ($Z \neq Z_0$)[10]. The top diagram is for an unbalanced configuration and the bottom is for a balanced configuration. ρ is the reflection coefficient, V_R and V_F are the reverse and forward waves respectively, each considered to be sinusoidal. All three parameters are expressed in magnitude and phase.

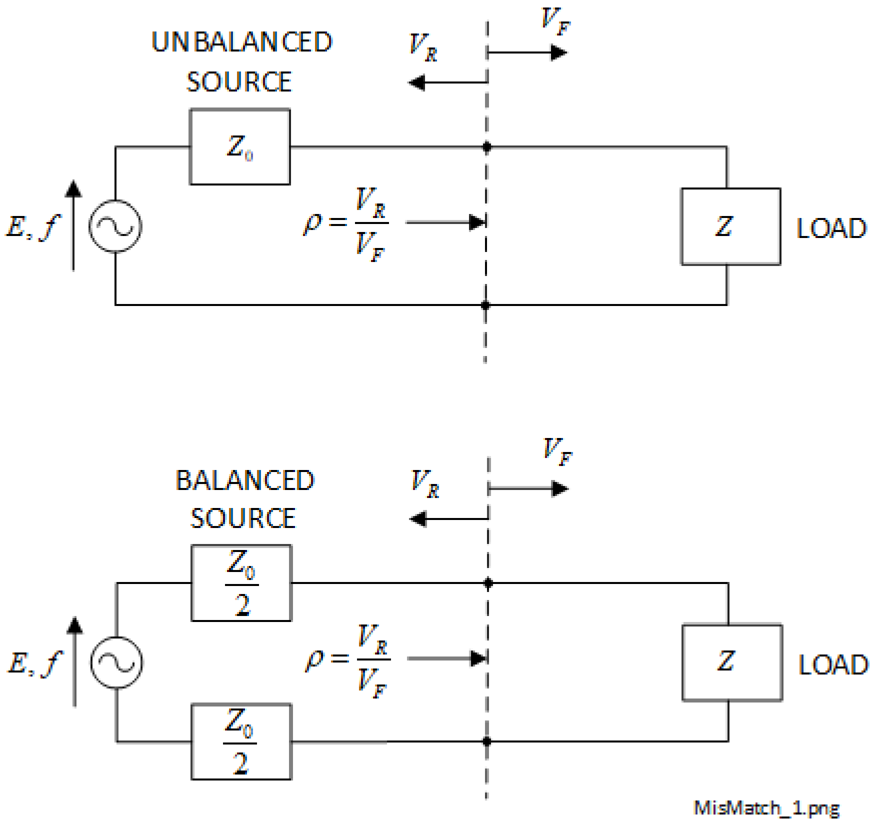


Figure 3-1 Circuit schematics for the connection of an unbalanced and a balanced Thevenin equivalent source of impedance of Z_0 to a load Z ($Z \neq Z_0$). The operating frequency wavelength is much greater than the circuit dimensions so spatial phase is neglected.

The unbalanced source does not require to be grounded but typically, for coaxial connections, it will be. In the common coaxial excitations using $Z_0 = 50\,\Omega$ or $Z_0 = 75\,\Omega$, the coaxial shield or screen is connected to ground at the equipment interfaces. However, the balanced connections shown in the bottom diagram has become very popular in recent years due to its electromagnetic compatibility (EMC) advantages. The Smith Chart

[10] A traditional circuit schematic represents lumped element conditions unless function(s) of spatial phase are explicitly shown.

processing for either type is unchanged. The case for distributed element conditions, where the spatial phase is significant, is shown in Figure 3-2.

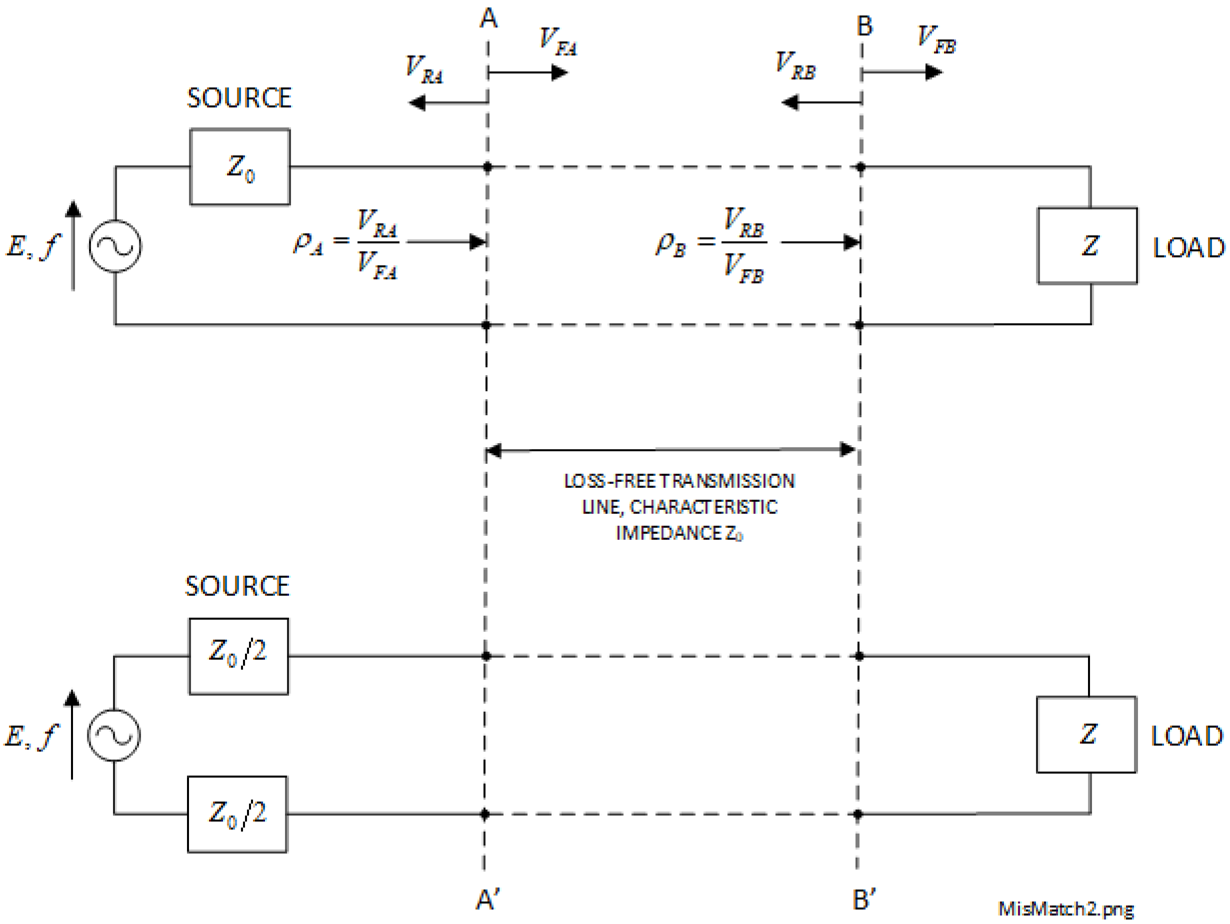


Figure 3-2 Circuit schematic, under distributed element conditions, for an unbalanced source (top) and balanced source each of impedance Z_0 connected via a length of loss-free transmission line of characteristic impedance Z_0 to an imperfect load of impedance Z ($Z \neq Z_0$). The Smith Chart circumferential (wavelength) scaling may be used in both cases to account for the effects of the spatial phase shift between the electrical planes AA' and BB'.

3.1.1 Multiple Reflections and the Quarter Wave Transformer [19] [22]

In both of the cases shown in Figure 3-2, forward sinusoidal signals from the source create travelling waves towards the load (left to right). Suppose that the impedance at AA' is Z_A and the impedance at BB' is Z_B ($Z_B = Z$). Both of these interfaces will cause reverse wave reflections (right to left). Unlike with the lumped element cases shown in Figure 3-1 for which there was only one reflection, this case will suffer a second reflection. Both reflections will affect both the forward and reverse waves. Therefore, under steady state conditions, there will be multiple reflections in both directions [19]. We will assume that the impedance mismatch will be small (Z is not close to an open or short circuit) so that, at each interface, the reflected wave will be much smaller than the incident wave and of negligible consequence[11]. A practical transmission line, with some loss, would diminish both waves even further.

Special cases occur when the wavelength is such that the length of the transmission line is either an even number of quarter wavelengths, or an odd number of quarter wavelengths as summarised below.

[11] In the other extreme, a perfect resonator with short or open terminations could not couple any power.

- The Transmission Line is an Even Number of Quarter Wavelengths [19] [27] [28] [29]

Each end of the transmission line *considered in isolation* would either be a short circuit or an open circuit. In the actual circuit, these are shunted by the resistive parts of the source impedance at the left and the load impedance at the right. This is a classic resonator condition for a distributed circuit, analogous to a series or parallel L-C circuit in the lumped element case [19][12]. Across a range of frequencies it would display a bandpass filter effect from the source to the load. However, the source and load impedances must be high (lightly loaded) for narrow band operation.

- The Transmission Line is an Odd Number of Quarter Wavelengths [19] [27] [28] [29]

This condition creates a quarter wave transformer, with the narrowest bandwidth for a single quarter wavelength and wider bandwidth for higher orders. Quarter wave transformers are distributed components which behave similarly to traditional (lumped element) transformers provided the correct conditions are met for the source and load and the impedance of the line itself.

3.2 Characteristic Impedance and Characteristic Admittance

The TLEs applied to a real, uniform transmission line, one with finite loss, derive (3.1) for the characteristic impedance (or system impedance) Z_0 [9] [11] [12].

$$Z_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}} \quad (3.1)$$

The real transmission line parameters are:

R, the series resistance per unit length in ohms per metre (Ω / m);
G, the shunt conductance per unit length in siemens per metre (S / m);
L, the series inductance per unit length in henrys per metre (H / m);
C, the shunt capacitance per unit length in farads per metre (F / m).
ω, the angular frequency in radians per second (rad / s).

For the loss free transmission line, $R = 0$ and $G = 0$. Therefore:

$$Z_0 = \sqrt{\frac{L}{C}} \quad (3.2)$$

The result of (3.2) is real only[13]. Although we can see from (3.1) that non-real Z_0 values are possible, practical transmission lines, are designed by the manufacturer with L and C values to give a Z_0 result very close to a real value such as 50 Ω or 75 Ω across a specified operating frequency range. Z_0 is *defined* as a real (resistive) impedance. Alternatively, we could have used the equivalent *characteristic admittance* Y_0 instead (3.3) but this is less commonly used. The SI unit for characteristic admittance is the siemen (S).

$$Y_0 = \frac{1}{Z_0} = \sqrt{\frac{C}{L}} \quad (3.3)$$

---

[12] The series L-C is low impedance and the parallel L-C is high impedance. The equivalent could be designed in high frequency lumped elements across a range of frequencies.

[13] Ground penetrating radar (GPR) involves processing intrinsic impedance (the equivalent of Z_0) which is not real.

After a manufacturer designs a cable, the resulting characteristic impedance for a prototype cable is measured accurately at increasing frequencies. If the value of Z_0 develops into significant non-real components, then the maximum specified frequency is reduced accordingly[14].

3.3 Balanced and Unbalanced Transmission Lines

Schematics for unbalanced and balanced connections including transmission lines are shown in Figure 3-1 and Figure 3-2. Coaxial cables are still widely used which, by definition, are unbalanced. Signals are measured with respect to ground, sometimes referred to as common mode (CM) excitation.

One disadvantage of CM excitation is that it is susceptible to CM interference and will cause CM emissions, making it difficult to meet electromagnetic compatibility (EMC) requirements [23]. Many high speed baseband data transmission lines such as Ethernet and Gigabit are balanced, excited in differential mode (DM). EMC (radiation and susceptibility) are less challenging using DM compared to CM. Balanced transmission lines have been developed accordingly as shown in the bottom schematic of Figure 3-2. In transmission line theory, although there are physical differences between balanced and unbalanced constructions, the transmission line derivations are unchanged[15].

3.3.1 Why Do We Assume the Transmission Line is Loss-Free?

A very reasonable question might be, “why do we assume the transmission line is loss-free when we know that practical transmission lines are not loss-free”?

All of the TLEs may be represented in lossy form but it gets very heavy mathematically and beyond the scope of this article which is aimed at the Smith Chart. To correctly represent lossy lines on a Smith Chart would disproportionately clutter it with extra scaling which may reduce its usefulness for real world problems. It is common practice to address any issues of cable loss separately from the Smith Chart analyses.

For lumped element conditions, we can still represent imperfect (lossy) lumped elements, as they will have finite resistive components which will probably also be functions of frequency. A better solution might be to measure them using a vector network analyzer (VNA) at the required frequencies with a suitable jig and calibration kit.

For distributed element conditions, the effects of longer component interconnections, such as cables and long PCB tracks become significant. We can only use the circumferential Smith Chart scaling if we are representing a near loss-free transmission line with the same characteristic impedance that is being used for the Smith Chart. This situation is shown in Figure 3-2. The two reference planes between which the Smith Chart may be used are AA’ and BB’. The transmission line between them is assumed to be: (a) loss-free and (b) to have a characteristic impedance of Z_0. In many cases these assumptions will be acceptable. The Smith Chart will give a quick result, perhaps not with the accuracy of a good CAD application, but showing if the design is progressing in the right direction, in time for corrective actions if required. As the project develops and we get closer to production, we can replicate data to CAD applications and refine the design.

3.4 Normalized Impedance and Normalized Admittance

Normalized impedances and normalized admittances use the references of characteristic impedance (Z_0) and characteristic admittance (Y_0) respectively. These may alternatively be described as system impedance and system admittance. The most commonly used characteristic impedance is 50 Ω, which is equivalent to a system admittance of 0.02 siemens (S). Z_0 and Y_0 are defined values. As they are non-reactive, we could refer

[14] In general, as the frequency is increased, the transverse cable dimension must be reduced maintain a resistive Z_0. Unfortunately this also reduces the power rating.

[15] Practical balanced (DM) transmission line cables tend to have higher Z_0 values, typically 100 Ω to 300 Ω.

to them as a resistance or conductance respectively, but it is common practice to still call them impedances or admittances. They have a reciprocal relationship:

$$Y_0 = \frac{1}{Z_0} \tag{3.4}$$

The most commonly used Smith Chart is the normalized impedance or Z type. In many Smith Chart problems we also need to use it in normalized admittance or Y type, perhaps converting between both several times before we arrive at the answer[16]. One option is to use a ZY Smith Chart and use either the Z or Y scale as appropriate, but the dense scaling may be difficult to read. An alternative is to use the same scales but perform a transformation in and out of Y scaling mode when required. Any point on a Z Smith Chart may be readily converted to the equivalent on a Y Smith Chart by a simple translation: 180° rotation of the associated voltage reflection coefficient vector about the Smith Chart center ($z = 1 \pm j0$) without changing its magnitude. Then the same scaling is used until the next transformation, but interpreting it as normalized admittance instead of normalized impedance. Exactly the same transformation may be applied for converting back from the Y to Z scaling, this time the Smith Chart center being at $y = 1 \pm 0$.

It is common practice to use a lower case 'z' (z) for normalized impedance and a lower case 'y' (y) for normalized admittance. Normalized values like these have no units but are still linear vectors so have values of magnitude and phase. They are expressed in complex rectangular or complex exponential form, thus obeying the rules of complex algebra and Euler's equations [20].

Normalised impedance and normalised admittance are related to (absolute) impedance and (absolute) admittance (upper-case Z and Y respectively) by the following equations[17]:

$$z = \frac{Z}{Z_0} \tag{3.5}$$

and

$$y = \frac{Y}{Y_0} \tag{3.6}$$

An example of a z to y transformation using the Smith Chart is shown in Figure 3-3. Vector OP is a normalised impedance of $z = 1.40 + j1.30$. This may be confirmed by reading off the normalized resistance and normalized reactance scaling. To move to the equivalent normalised admittance the user may draw a circle centered at $z = 1 \pm j0$, passing through P and then move the vector OP through 180° to OQ. All scaling until the next similar transformation is now in normalised admittance. Reading from the Smith Chart scaling at point Q now indicates a normalised admittance of $y = 0.38 - j0.36$. A similar process will transform from y back to z remembering that, in this case the Smith Chart center O will be at $y = 1 \pm j0$. Some elementary complex number algebra will confirm this.

[16] In describing Smith Charts, Z or Y tends to be upper case even though normalized values are used.

[17] For normalized values we will always use the adjective 'normalized'. Otherwise, assume the values are 'absolute' even if this adjective may sometimes be omitted.

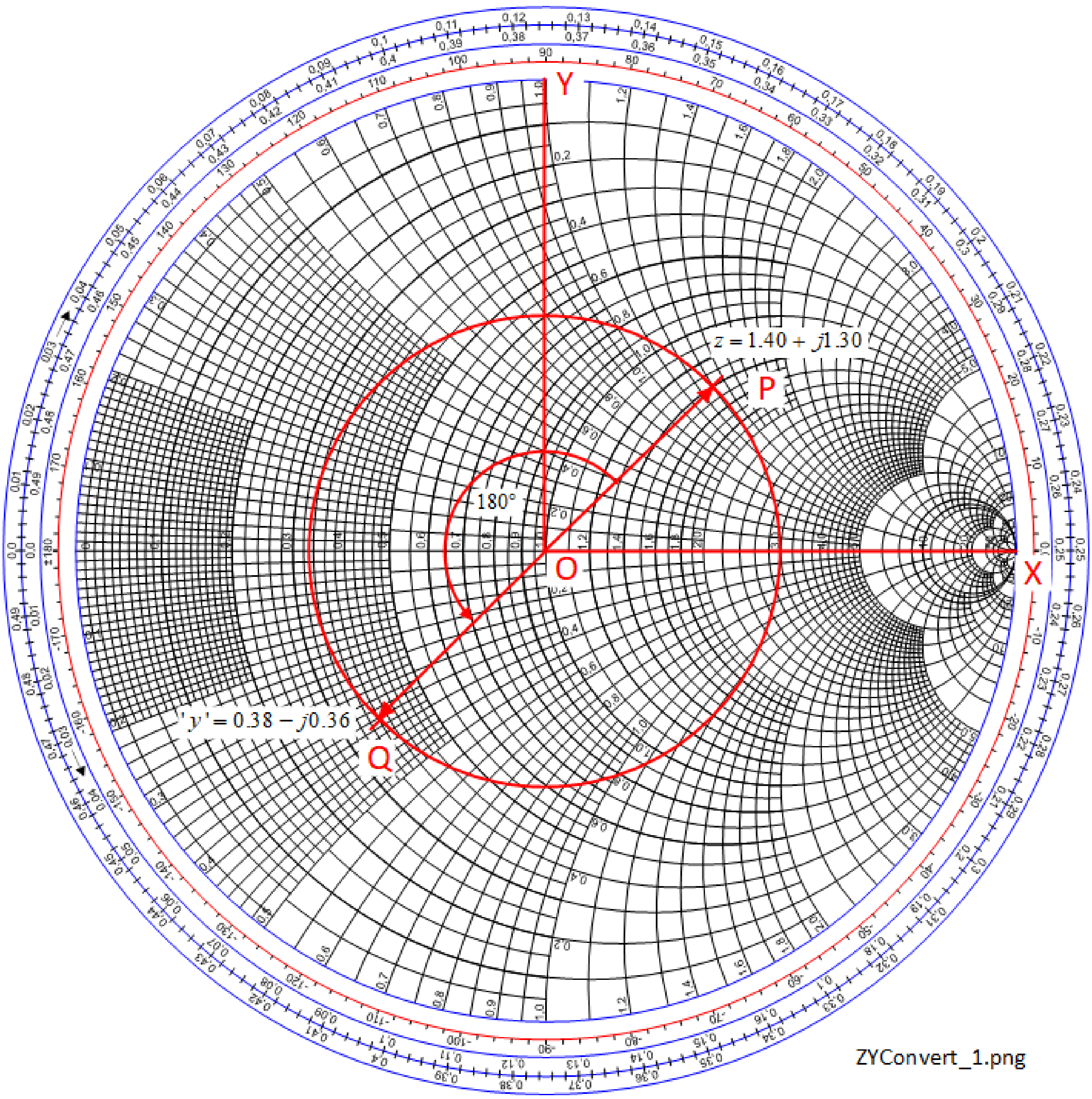


Figure 3-3 An illustration of how a point P on a Z Smith Chart may be converted to point Q, the equivalent in the normalized admittance, or Y Smith Chart by rotating the corresponding reflection coefficient vector through 180˚ at constant magnitude.

3.5 Standing Waves and Voltage Reflection Coefficient

Standing waves occur on an imperfectly matched transmission line. Usually, it is required to reduce the reflected voltage component as far as possible so that it may be considered negligible.

3.5.1 Voltage Reflection Coefficient

We will refer again to the schematics for an imperfectly matched line for lumped and distributed element conditions shown in Figure 3-1 and Figure 3-2 and respectively.

In both cases, the forward wave (left to right) expressed as an instantaneous (temporal and spatial) voltage is V_F. As the circuits are imperfectly matched, a similar finite but reflected wave exists (V_R) in the opposite direction. Using complex exponential notation according to general forms of Euler's equations [20]:

$$e^{jx} = \cos x + j \sin x \quad (3.7)$$

$$e^{-jx} = \cos x - j \sin x \quad (3.8)$$

Using complex exponential notation for V_F and V_R:

$$V_F = |V_F| \operatorname{Re}\left(e^{j(\omega t - \beta x)}\right) \quad (3.9)$$

$$V_R = |V_R| \operatorname{Re}\left(e^{j(\omega t + \beta x)}\right) \quad (3.10)$$

The symbols have the following meanings:

- $\operatorname{Re}(f)$ means 'real part of' the function f;
- the operator $|f|$ means 'magnitude of', or amplitude of the function f;
- $\beta = \frac{2\pi}{\lambda}$ is the spatial phase component at the frequency of operation in radians per metre (rad/m), where λ is the wavelength at the operating frequency within the transmission medium in metres (m);
- x is the direction of propagation along the transmission line in metres (m);
- $\omega = 2\pi f$ is the angular frequency in radians per second (rad/s), the time-dependent (temporal) phase component;
- f is the frequency in hertz (Hz);

For V_F and V_R, the sign in front of the β symbol uses the convention of left to right being the positive direction.

The voltage reflection coefficient ρ is defined as the ratio of the reverse to the forward voltage waves:

$$\rho = \frac{V_R}{V_F} \quad (3.11)$$

There also exists a *current* reflection coefficient but here we will use the voltage version even if we do not always explicitly use the adjective 'voltage'. That is because, in test equipment detectors, forward and reverse voltages are usually easier to measure on a transmission line than currents.

As both V_R and V_F are voltage vectors, ρ is also a vector expressed as a complex quantity but it is unitless. Provided that the system is operating normally without any instabilities, $|V_R| \le |V_F|$ and $|\rho| \le 1$. The case $|\rho| = 0$ is a perfect match when $Z = Z_0$. The case $|\rho| = 1$ is a perfect (100%) reflection such as the result of perfect short circuit or a perfect open circuit. Whilst $|\rho|$=1 for such an open or short circuit, the *phase* of ρ is $180°$ or $0°$ for each case respectively.

3.5.2 Standing Waves: Total Voltages and Currents

From Section 3.5.1 we noted that, for an imperfectly matched load at a fixed frequency f under steady state conditions, a voltage standing wave V will form which will be a function of both V_F and V_R. An associated current wave I will also form, again comprising functions of V_F and V_R. Using transmission line theory, these will be described by the following equations [9] [11] [12]:

$$V = V_F + V_R \tag{3.12}$$

$$Z_0 I = V_F - V_R \tag{3.13}$$

Dividing (3.12) by (3.13), will yield the normalized load impedance z which is the (absolute) load impedance Z divided by Z_0:

$$z = \frac{V}{Z_0 I} = \frac{Z}{Z_0} \tag{3.14}$$

Taking this further and using the definition of voltage reflection coefficient (3.11), then for normalised impedance:

$$z = \frac{Z}{Z_0} = \frac{V_F + V_R}{V_F - V_R} = \frac{1 + V_R/V_F}{1 - V_R/V_F} = \frac{1+\rho}{1-\rho} \tag{3.15}$$

A similar argument, but for normalised admittance y gives the following result.

$$y = \frac{1}{z} = \frac{Y}{Y_0} = \frac{1-\rho}{1+\rho} \tag{3.16}$$

Knowing Z_0 and therefore Y_0, we can convert ρ to or from any impedance Z or admittance Y.

3.5.3 The Variation of Voltage Reflection Coefficient with Position Along a Transmission Line

Until now we have considered *lumped element conditions*, so we have assumed any transmission lines to negligible electrical length. This is shown in Figure 3-1. Now we will consider distributed element conditions: transmission line lengths are assumed to be significant[18]. Refer to Figure 3-2 which shows two schematics for distributed element conditions: for an unbalanced line at the top and a balanced line at the bottom. The type of line does not matter but we are still assuming it to be loss-free.

As we discussed in Section 3.5.1, the voltage reflection coefficient (ρ) is used as a basis for transmission lines generally and the Smith Chart in particular. It is a complex quantity, the ratio of the reverse voltage (V_R) to the forward voltage (V_F). V_F and V_R both have System International (SI) units of volts (V) and may be represented as time and space varying voltage vectors[19]. Each may be represented in magnitude and phase using complex expressions. ρ is therefore also a linear complex expression but it is unitless.

We are using sinusoidal voltages and currents which have time varying (temporal) functions. These are also (spatial) functions of position, so the instantaneous value of V_F, V_R and therefore ρ are functions of both time and position. Using the Euler derived complex exponential notation which was described in Section 3.5.1, the generic expression for a time and space varying sinusoidal voltage wave in the forward direction (left to right) is:

[18] The transmission line electrical length is significant but still loss-free.
[19] Also known as phasors.

$$V_F = V_{F0}\,\mathrm{Re}\left[e^{j(\omega t-\beta x)}\right] \quad (3.17)$$

- V_{F0} is the amplitude (peak) voltage of the time varying sinusoidal (or co-sinusoidal) waveform in volts (V).
- $\mathrm{Re}(\ )$ is the 'real part of operator. In electrical engineering, it is common practice to omit this in the complex algebra with no mathematical consequence.
- ω is the angular frequency in radians per second (rad/s).
- t is the instantaneous time in seconds (s)
- β is the spatial phase constant in radians per metre (rad/m).
- x (lower case) is the distance along the positive direction of propagation, left to right.

We may further simplify ω and β as follows:

$$\omega = 2\pi f \quad (3.18)$$

$$\beta = \frac{2\pi}{\lambda} \quad (3.19)$$

- f is the temporal frequency in hertz (Hz).
- λ is the wavelength in the medium containing the electric field, the cable dielectric.

Each Smith Chart point is considered at one frequency to complete all the necessary graphical procedures[20]. The time varying part ($e^{j\omega t}$) of the voltage wave equation may therefore be omitted (replaced by unity) from the voltage wave equation, so (3.17) at that one frequency becomes:

$$V_F = V_{F0}\,\mathrm{Re}\left[e^{-j\beta x}\right] \quad (3.20)$$

Similarly, in the reverse wave expression, the sign of the exponent power is positive.

$$V_R = V_{R0}\,\mathrm{Re}\left[e^{j\beta x}\right] \quad (3.21)$$

Recall that the definition of voltage reflection coefficient (3.11) is:

$$\rho = \frac{V_R}{V_F} \quad (3.22)$$

Omitting the real part operator and substituting (3.20) and (3.21) into (3.22) for positions AA' and BB' as shown in Figure 3-2. This gives the following equations where subscripts have been added: A and B for positions A and B respectively and subscript '0' for the magnitude of the associated waveform:

$$\rho_A = \frac{V_{RA}}{V_{FA}} = \frac{V_{RA0}e^{j\beta x_A}}{V_{FA0}e^{-j\beta x_A}} \quad (3.23)$$

[20] Once completed, similar constructions may be performed at another frequency.

$$\rho_B = \frac{V_{RB}}{V_{FB}} = \frac{V_{RB0}e^{j\beta x_B}}{V_{FB0}e^{-j\beta x_B}} \quad (3.24)$$

For the reflection coefficient at AA' with respect to BB' then ρ_A must be divided by ρ_B:

$$\frac{\rho_A}{\rho_B} = \frac{V_{RA0}e^{j\beta x_A}V_{FB0}e^{-j\beta x_B}}{V_{FA0}e^{-j\beta x_A}V_{RB0}e^{j\beta x_B}} \quad (3.25)$$

This is where we appreciate using a loss-free transmission line. This means neither the *amplitudes* of the forward waves nor the amplitudes of the reverse waves are functions of position. Therefore $V_{RA0} = V_{RB0}$ and $V_{FA0} = V_{FB0}$, so these terms in (3.25) cancel out giving:

$$\rho_A = \rho_B e^{j2\beta(x_A - x_B)} \quad (3.26)$$

The *magnitude* of the reflection coefficient does not change between AA' and BB' but the phase does according to (3.26). x_A and x_B are the positions of the AA' and BB' respectively so $x_B - x_A$ is the distance between them. The factor of 2 in the exponent is because the phase of both the forward and reverse waves are functions of this distance.

3.5.4 Impedance Variation Along a Loss-Free Transmission Line [9]

In Figure 3-2, the positions of the ends of the transmission lines were at x_A followed by x_B in the forward direction. Therefore, if the length of the line is $l = x_B - x_A$, then from (3.26):

$$\rho_A = \rho_B e^{-j2\beta l} \quad (3.27)$$

Suppose the impedances at AA' and BB' are Z_A and Z_B respectively. From (3.12) and (3.13), the general impedance Z is given by:

$$Z = \frac{V}{I} = Z_0\left(\frac{V_F + V_R}{V_F - V_R}\right) = Z_0\left(\frac{1+\rho}{1-\rho}\right) \quad (3.28)$$

Therefore:

$$Z_A = Z_0\left(\frac{1+\rho_A}{1-\rho_A}\right) \quad (3.29)$$

$$Z_B = Z_0\left(\frac{1+\rho_B}{1-\rho_B}\right) \quad (3.30)$$

Re-arranging (3.30) in terms of ρ_B gives:

$$\rho_B = \frac{Z_B - Z_0}{Z_B + Z_0} \quad (3.31)$$

Substituting from (3.26) into (3.29):

$$Z_A = Z_0\left(\frac{1+\rho_B e^{-j2\beta l}}{1-\rho_B e^{-j2\beta l}}\right) = Z_0\left(\frac{e^{j\beta l}+\rho_B e^{-j\beta l}}{e^{j\beta l}-\rho_B e^{-j\beta l}}\right) \quad (3.32)$$

Substituting from (3.31) into (3.32):

$$Z_A = Z_0\left(\frac{(Z_B+Z_0)e^{j\beta l}+(Z_B-Z_0)e^{-j\beta l}}{(Z_B+Z_0)e^{j\beta l}-(Z_B-Z_0)e^{-j\beta l}}\right) \quad (3.33)$$

Euler's equations, (3.7) and (3.8), may be used to convert (3.33) to its real rectangular form:

$$Z_A = Z_0\left(\frac{Z_B\cos\beta l + jZ_0\sin\beta l}{Z_0\cos\beta l + jZ_B\sin\beta l}\right) = Z_0\left(\frac{Z_B + jZ_0\tan\beta l}{Z_0 + jZ_B\tan\beta l}\right) \quad (3.34)$$

Referring to Figure 3-2, (3.34) provides an equation to calculate the impedance at AA' given the impedance at BB' for the loss-free transmission line of length l and characteristic impedance of Z_0.

The normalized version of (3.34) for z_A (noting the lower case symbol) is:

$$z_A = \frac{Z_A}{Z_0} = \frac{\frac{Z_B}{Z_0} + j\tan\beta l}{1 + j\frac{Z_B}{Z_0}\tan\beta l} = \frac{z_B + j\tan\beta l}{1 + jz_B\tan\beta l} \quad (3.35)$$

This may be useful for direct interaction with the Smith Chart, using the same impedance as the cable.

Each of the angle function arguments (βl) is the product of the spatial phase constant, β in radians per metre (rad/m) and the length of the transmission line in metres (m), so it is normally expressed in radians. Often β is simply described as 'phase constant' with the understanding that it is actually the spatial phase constant, defined as:

$$\beta = \frac{2\pi}{\lambda_m} \quad (3.36)$$

λ_m is the wavelength in the transmission line itself, as opposed to, say, free space. A practical transmission line requires at least some form of insulator mounts to separate the conductors and typically substantial air spacing. The *bulk* dielectric constant (or relative permittivity) of the insulator material will differ from the *effective* dielectric constant of the combination, say ε_{eff}, normally available from the cable manufacturer. The value of ε_{eff} determines the speed of propagation through the cable v, where:

$$v = \frac{c}{\sqrt{\varepsilon_{eff}}} \quad (3.37)$$

Then λ_m is obtained from:

$$\lambda_m = \frac{v}{f} = \frac{c}{f\sqrt{\varepsilon_{eff}}} \quad (3.38)$$

3.5.5 Time Independence

The transmission line equations for the forward and reverse waves, (3.9) and (3.10) respectively, include both the temporal phase component ($e^{j\omega}$) and the spatial phase component ($e^{-j\beta x}$ or $e^{j\beta x}$). As we are considering one frequency at a time, for each set of Smith Chart calculations, the associated temporal value, or angular frequency ω is constant and will therefore cancel out and be omitted from the calculations. However, the impedances of the lumped element components (capacitors and inductors) will still be functions of the chosen working frequency, so this must be used to extract high frequency data from the manufacturers' data.

3.5.6 Return Loss and Voltage Standing Wave Ratio

Return loss (RL) and voltage standing wave ratio (VSWR, s) are common scalar measures of the quality of electrical match of a load to the characteristic impedance. Both parameters are functions of the *magnitude* of the voltage reflection coefficient (3.11) according the following equations:

$$RL = -20\log_{10}|\rho| \quad dB \tag{3.39}$$

$$s = \frac{|V_F| + |V_R|}{|V_F| - |V_R|} = \frac{1+|\rho|}{1-|\rho|} \tag{3.40}$$

The negative sign on the right hand side of (3.39) is included because this parameter is defined as a loss rather than a gain. RL is a logarithmic measure, normally expressed in decibels (dB). VSWR, represented by lower case 's', is a linear measure which is often expressed as a unitless ratio relative to unity, for example 1.22:1 which is equivalent to a return loss of 20 dB[21]. RL benefits from integrating easily with other popular logarithmic measures such as dBm and dBW for powers. VSWR is often adopted on high power transmission lines, for example at the connection of a transmitter to an antenna, where the linear scaling for VSWR may be more appropriate. Table 3-1 lists some values of equivalent return loss, reflection coefficient magnitude and VSWRs.

Table 3-1 Some example values of return loss, reflection coefficient magnitude, VSWR and transmission power ratios. VSWR is typically used in high power applications and return loss is popular at low powers, especially associated with vector network analyzers (VNAs).

Return Loss (dB)	Reflection Coefficient Magnitude $\|\rho\|$	VSWR (s:1)	Transmitted Power Relative to Incident Power (P_T/P_I)
1.0	0.891	17.39	0.206
3.0	0.708	5.85	0.499
6.0	0.501	3.01	0.749
10.0	0.316	1.92	0.900
20.0	0.100	1.22	0.990
30.0	0.032	1.07	0.999

In general, at low powers, the reason for requiring a good match (high return loss) at electrical interfaces is to enable the equipment to perform within specification across the operating frequency range. Important parameters to measure in order to achieve this include gain, return loss, slope, ripple, stability. The risk of a poor match is to degrade performance. It is unlikely to be hazardous or cause any damage, but it might be causing or susceptible to electromagnetic interference. At high power levels the risks of a poor match are

[21] A perfect match has a VSWR of 1:1 but a RL of a 'large number' in dB. So VSWR is often used where the consequences of an excessive reflection is more serious, such a high power feed to a transmitting antenna.

much greater: a possible non-ionising radiation hazard, excessive dissipation, possible damage and wasted power.

4 Construction and Use of the Smith Chart [1] [7] [8] [10] [14] [16]

In this chapter we will bring together the introduction to Smith Charts in Chapter 2 and the transmission line equations in Chapter 3 to describe the construction of the Smith Chart and how it is used. An example of a blank Smith Chart is shown in Figure 1-1 and a partially annotated Smith Chart in Figure 3-3, including the superimposed, rectangular x and y axes. As we noted in Chapter 2, we will be using the Smith Chart only inside the ‘unity radius’ region, where the magnitude of the reflection coefficient is less than or equal to 1, $|\rho| \le 1$.

4.1 Construction of the Smith Chart

Figure 4-1 shows an annotated normalized impedance (Z) Smith Chart including an arbitrary ρ vector OP. The scaling is linear, in normalised impedance, superimposed on the polar diagram of the reflection coefficient (ρ) also represented in linear polar form. The reflection coefficient scaling is not shown but is implicit from the geometry. This is the most common orientation with the rectangular x-axis and y-axis represented as straight horizontal and vertical lines respectively. Positive phase angles are defined as counter-clockwise starting at zero degrees relative to the x = axis.

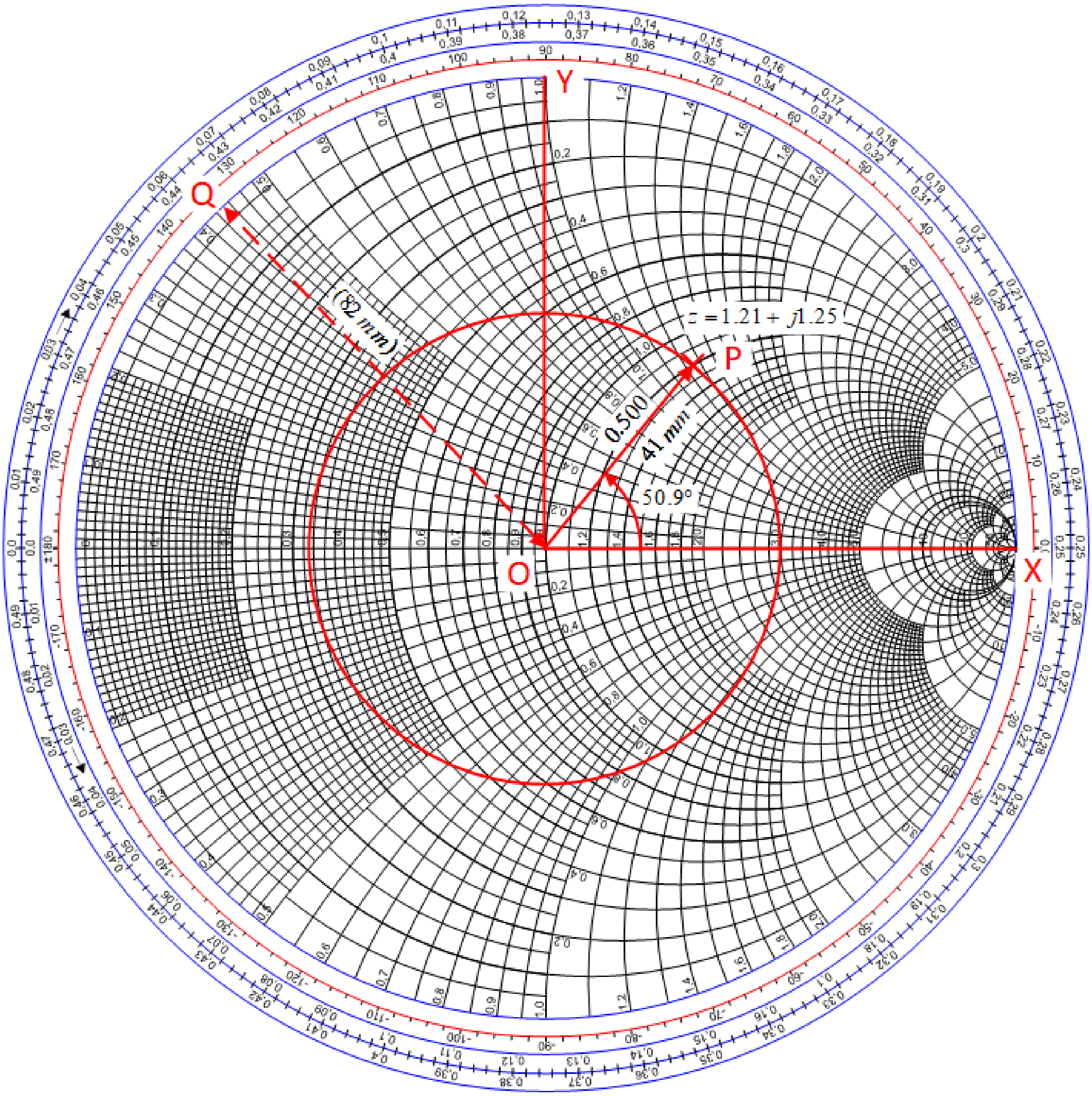


Figure 4-1 A normalized impedance (Z) Smith Chart example showing how a value of complex reflection coefficient ($\rho = 0.500\angle 50.9°$) may be readily converted to the equivalent value of normalized impedance ($z = 1.21 + j1.25$).

Using our definition, the angle shown ($50.9°$) is equivalent to the phase of ρ. The vector magnitude is 0.5 ($|\rho| = 0.5$). This is scaled linearly from the unity radius circle ($|\rho| = 1$) to the center point which is effectively a another circle of zero radius. The magnitude and phase of OP may be represented in polar form as $\rho = 0.5\angle 50.9°$. Alternatively, its rectangular form would be $\rho = 0.315 + j0.389$. The TLE (3.15) relates normalized impedance (z) to reflection coefficient (ρ)[22]. Substituting for ρ and a little complex arithmetic will confirm the result $z = 1.208 + j1.254$.

[22] This equation is independent of Z_0

As we have discussed, the scaling of the reflection coefficient polar diagram covers the range $|\rho| = 0$ (a point at the center) to $|\rho| = 1$). $|\rho| = 0$ is a perfect match (a circle of zero radius, or a point) and $|\rho| = 1$ is a 100% reflection (a perfect open circuit or short circuit).

4.2 Regions of the Smith Chart

The regions of the Smith Chart for normalized impedance (z) and normalized admittance (y) are shown in Figure 4-2. In each case, the positive reactive parts are located in the blue area and the negative reactive parts are located in the green area. We will continue to use upper case symbols for absolute values (with appropriate SI units) and lower case symbols for normalised units (which are unitless).

If the normalized real and imaginary components of resistance and reactance are r and x respectively, then:

$$z = r \pm jx \qquad (4.1)$$

Similarly, if the normalized components of conductance and susceptance are respectively g and b , then:

$$y = g \pm jb \qquad (4.2)$$

Taking the normalized impedance equation (4.1) as an example, it can be seen from Figure 4-2 that the real and complex coefficients are scaled into lines of constant r and lines of constant x . Lines of constant r are complete circles with centers along the x axis from $r = \infty$ on the right to $r = 1$ at the center. The radii of those circles start at zero at $r = \infty$ to a maximum at $r = 1$ which is coincident with the $|\rho| = 1$ circle. Lines of constant x form incomplete arcs of circles and comprise sets of positive coefficients and sets of negative coefficients.

Similar arguments apply to the Smith Chart using the normalized admittance equation (4.2): normalized resistance replaced by normalized conductance and normalized reactance replaced by normalized susceptance.

As discussed in Section 3.4, for absolute or un-normalized versions of the same equations the symbols used will be in upper case, showing the units as follows:

$$Z = R \pm jX \quad \Omega \qquad (4.3)$$

$$Y = G \pm jB \quad S \qquad (4.4)$$

These would be used when we wish to convert in or out of the element units: resistance, capacitance or inductance.

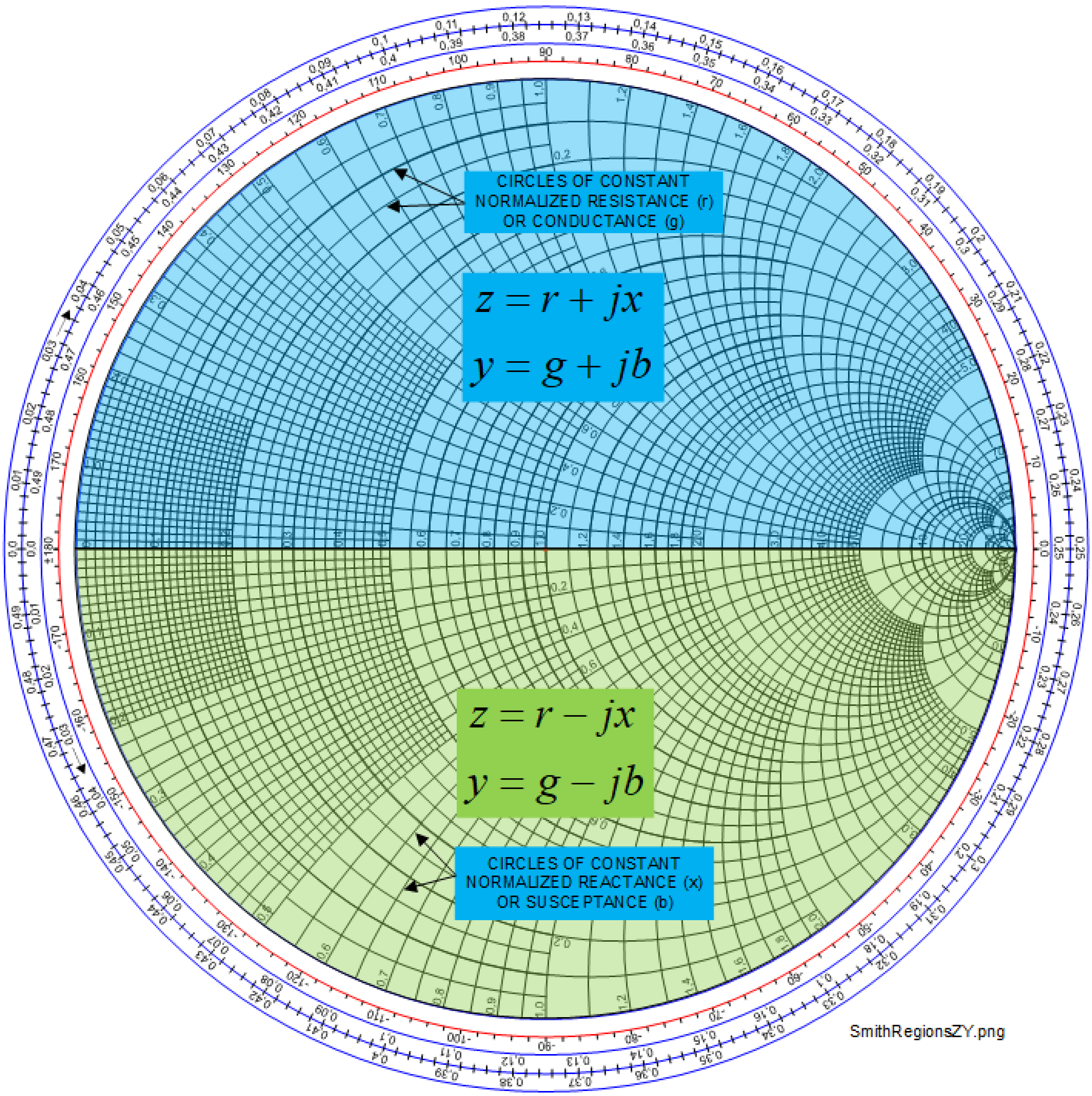


Figure 4-2 Regions of the Smith Chart used to represent normalized impedance (z) and normalized admittance (y). The scaling lines comprise lines of constant normalized resistance (or conductance) and lines of constant normalized reactance (or susceptance).

4.3 Using the Smith Chart

4.3.1 How Accurate is the Smith Chart?

Referring to the example Smith Chart shown in Figure 4-2, each of the scaled ranges mathematically covers zero to infinity, but it is not realistic above about 20. Therefore, we wish to avoid the large number regions as far as possible due to the limitations for extrapolation. The closer plotted values and measurements are to the center the better. Using the definition of return loss in Section 3.5.6, the area inside the 10 dB return loss circle ($|\rho| = 0.316$) should be taken as the worst case, but 20 dB return loss circle ($|\rho| = 0.100$) would be better.

These areas represent relatively well matched components. A common mistake using Smith Charts is to assume almost anything can be matched to a common impedance like 50 Ω.

4.3.2 What is a Suitable Choice for Z_0?

The most common nominal value of characteristic impedance (Z_0) is 50 Ω. For many years this has been used widely for equipment interfaces in the form of coaxial cables, connectors and microstrip lines[23]. Ultimately, Z_0 values have evolved from practical cable design and 'convenient' values to interface equipment sources and loads such as signal generators and antennas[24]. More recent cable developments for the transmission of high speed data over balanced lines, such as Ethernet and Gigabit using differential excitation, has tended to slightly higher impedances. Other common examples are 75 Ω, 90 Ω, 100 Ω, 120 Ω, 300 Ω and 600 Ω.

4.3.3 Do I Start in Normalized Impedance or Normalized Admittance Mode?

Whatever is most appropriate. If the first connection is expected to be in series, use impedance or in parallel use admittance. Following on from this, Figure 4-3 shows all the possible 'L' format matching circuits using two reactive elements, combinations of capacitors and/or inductors [13] [24].

[23] Coaxial implies unbalanced excitation and loading since the shield (screen) is grounded but this is immaterial to the TLEs.
[24] The impedance of a half wave dipole is about 73 Ω. Ultimately this was affected by factors including: size, cost, power handling, connector availability etc.

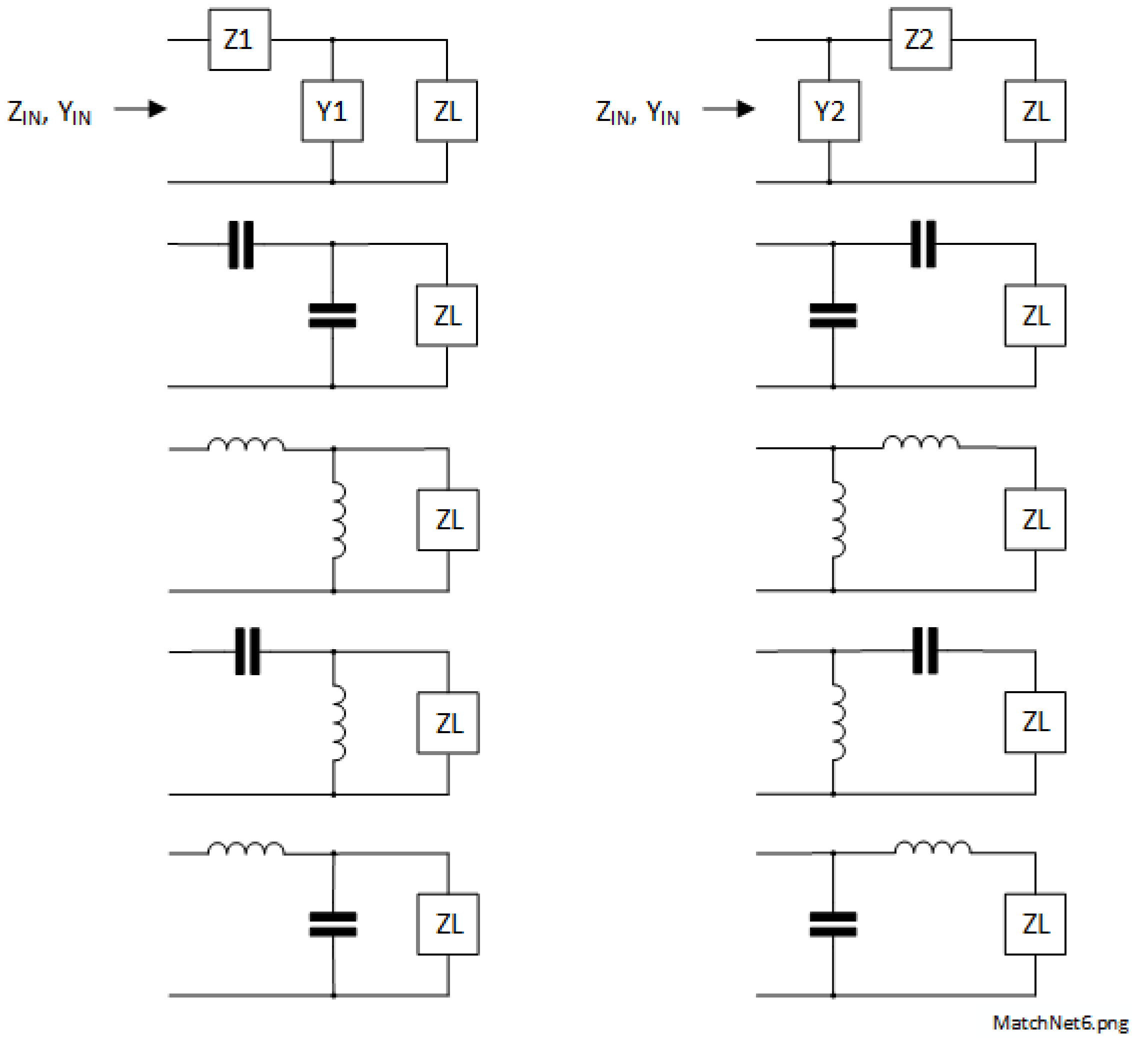


Figure 4-3 Alternative 2-stage reactive matching networks.

Working back from the load in each case, those on the left start with parallel reactances (admittances), followed by series reactances (impedances). The reverse applies for those on the right. Therefore, the Smith Chart impedances must be chosen accordingly.

If the normalized impedance of the load ZL is inside the $1 \pm jx$ constant resistance circle, then one of the matching networks shown on the left of Figure 4-3 must be chosen otherwise one on the right must be chosen [24]. There is then usually more than one option of matching the circuit. Some guidelines for matching elements are listed below, assuming the use of good quality reactive components appropriate to the chosen frequency range:

- If possible, capacitors should be chosen in preference to inductors as they achieve higher Q factors.
- Capacitors are usually cheaper than inductors.
- Capacitors are less prone to value drift than inductors.
- Where a capacitor and an inductor are required, try to use a smaller value inductor in preference to a smaller value capacitor.
- For matching at higher powers, take care to comply with the RF voltage rating and power dissipation requirements.

4.3.4 Can We Use Lumped Element Approximations?

Provided the shortest wavelength, at the highest operating frequency, is much longer than the basic dimensions of the circuit we are developing, including cables and connectors, it will be sufficiently accurate to use ‘lumped element’ conditions [24]. Lumped element conditions means that we are neglecting the effects that spatial propagation (physical positioning of the wave along a line) has on the phase of the signal. In fact, the majority of RF circuits are designed assuming lumped element conditions.

The shortest wavelength criterion relates to the propagation wavelength within the transmission line, as opposed to free space. As we discussed in Section 3.5.4, it must include possible corrections for the effective dielectric constant (ε_{eff}) of transmission lines which are part of the circuit. This leads to the following equations for the propagation velocity in the transmission line (v) and the associated wavelength (λ_m):

$$v = \frac{c}{\sqrt{\varepsilon_{eff}}} \tag{4.5}$$

$$\lambda_m = \frac{v}{f} = \frac{c}{\sqrt{\varepsilon_{eff}}\, f} \tag{4.6}$$

c is the speed of electromagnetic radiation in free space (m/s) and f is the frequency (Hz). Therefore the wavelength in the medium will be shorter than the wavelength in free space. If the lumped element criteria is met, the outer circumferential scaling of the Smith Chart for spatial phase corrections will not be used.

4.3.4.1 Secondary Characteristics of Lumped Element Components

Once we have decided on the ‘lumped element’ approximation we still need to make allowances for the high frequency, or ‘secondary’ , characteristics of lumped element components. Secondary characteristics are usually strong functions of frequency which make the performance of the component element (resistance, inductance or capacitance) non-ideal and are observed even at quite low frequencies[25]. Some of these phenomena are summarized below:

- Skin Effect

Even for operating frequencies as low as those widely used for power transmission (50 Hz and 60 Hz), skin effect can rarely be neglected.

- Finite Resistance of Reactive Components

Nominally ideal inductors and capacitors include wire coils and electrodes respectively, constructed from electrical conductors such as copper. These have finite resistances, also functions of frequency due to the skin effect, which may be significant. The apparent values will also be functions of frequency. For example, a larger inductance will generally have more turns to fit into a small size and a larger value capacitor may have more plates and therefore more connections.

- Inter-Winding Capacitance

Inductor coil windings may be closely spaced and possibly layered to achieve the required inductance. This construction increases interwinding capacitances within what is intended to be a near-ideal inductor. These may resonate with inductive elements to cause self resonant frequencies (SRFs).

- Self-Resonant Frequencies

Elements of inductance and capacitance in the same component potentially cause self-resonant frequencies (SRFs). SRFs usually start occurring beyond the high frequency end of the component’s normal frequency range. They may be recognised from the manufacturers’ datasheets when the trajectories of parameters such

[25] For example, skin effect limits the conductor thickness even at power frequencies (50 Hz or 60 Hz) .

as Q factor, apparent inductance and capacitance become discontinuous. These may not be explicitly described, even if they are beyond the maximum operating frequency. They may not exist or be as serious in a competitor's product. Using operating frequencies close to SRFs must be avoided as the resonances are unstable and unpredictable so operating the components well below any SRFs would normally produce more predictable and repeatable performance. Any case of an SRF being 'conveniently located', perhaps to suppress an unwanted spurious or similar, should be treated as abnormal and probably unrepeatable.

- Capacitor Dielectric Loss

An ideal capacitor dielectric is a perfect insulator. A perfect insulator is loss-free. Practical capacitor dielectrics, especially the high dielectric constant types, will become progressively more lossy as the operating frequencies are increased. This phenomenon will degrade the overall capacitive performance. If the RF rating is inadequate, the finite loss may cause excessive dissipation, reliability issues and possible failure.

4.3.4.2 High Frequency Performance: Inductors

Many manufacturers of lumped element components intended for high frequencies have developed design methods to minimise secondary characteristics. Due to the investment required in development, such parts are generally of higher cost than low frequency versions but usually include significant, accurately measured or simulated, design data. Inductors have a rich set of secondary characteristics. Some examples from Coilcraft®, a specialist manufacturer of high frequency parts, are shown in Figure 4-4 for their 0806SQ range of high frequency air core inductors [3].

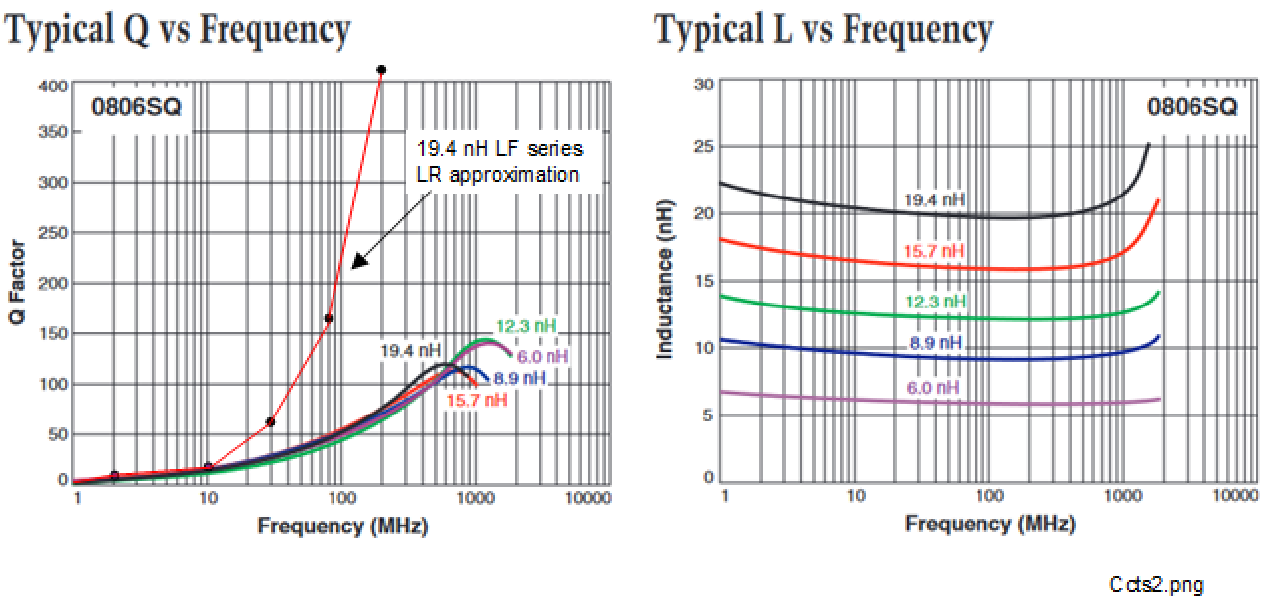
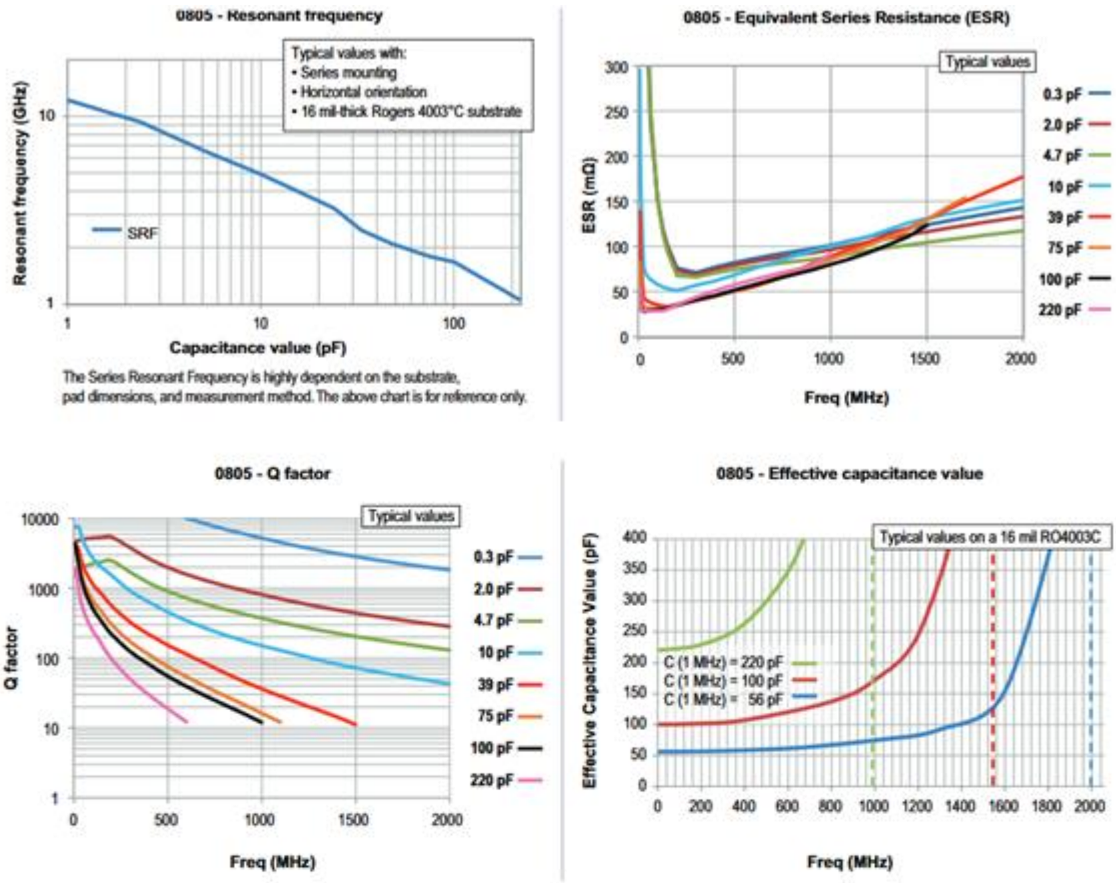


Figure 4-4 Example high frequency (measured or accurately simulated) design data from Coilcraft for their 0806SQ range of air core inductors. The Q factor plots include a prediction if a simple LF (series LR) model was assumed for the 19.4 nH inductor.

4.3.4.3 High Frequency Performance: Capacitors

Using the formula for inductive reactance included in Table 4-1, the impedance of an ideal 19.4 nH inductor, like the example in Section 4.3.4.2 at 100 MHz, is about 12 Ω. An example of a capacitor of about the same size with a similar impedance magnitude at 100 MHz is the Johanson® QSCT 0805 100 pF part [4]. A copy of the datasheet which includes this is shown in Figure 4-5. How can we relate this performance to that of the inductor?



Johan2.png

Figure 4-5 Manufacturer's data for Johanson High-Q chip capacitors type QSCT 0805.

For the capacitor data we can see immediately that the Q-factor values are much greater than those for the inductor. In fact, it is close to an ideal capacitor up to nearly 1 GHz[26]. The graphs of ESR (equivalent series resistance) against frequency imply that these are based on the R-C equivalent series circuit as shown in Figure 4-6 (c), as opposed to the parallel one. Using the formula provided, $Q = 1/(\omega C R_S)$, at 100 MHz, gives an ESR of about 0.02 Ω[27]. Furthermore, a few similar calculations at higher frequencies will demonstrate that the Q factors track reasonably well.

4.3.5 How Useful are Low Frequency Approximations?

Referring to the Coilcraft example inductors shown in Figure 4-4 we can see how the Q factor and measured (effective) inductances vary with frequency. Unfortunately it is difficult to accurately model these components and very often we have to use the data supplied by the manufacturer. If this data is not available for our desired frequency range, the part is probably designed for lower frequencies and will not behave well for our application. Taking the 19.4 nH inductor as an example, such a strange nominal value is probably a sort of average of the measured inductance over a range of frequencies. In an effort to model it, we might choose the rather simplistic model of a series inductor-resistor combination like that shown in Figure 4-6(a). The Q factor (Q) of such a series L-R circuit is given by the ratio of the magnitudes of the reactive (inductive) *voltage* (V_L) to the resistive *voltage* (V_R):

---

[26] The curve probably stops at about 1 GHz due to SRF issues.

[27] For the parallel equivalent circuit, R_P is about 12.7 kΩ.

$$Q = \frac{V_L}{V_R} = \frac{\omega L}{R_S} = \frac{2\pi fL}{R_S} \tag{4.7}$$

Where ω is the angular frequency in radians per second (rad/s), f is the temporal frequency (Hz) and R_S is the *series* inductor resistance in ohms (Ω). Taking a relatively low frequency of 10 MHz, inductance (L) of 20.5 nH and Q = 20 then, using (4.7) gives $R_S = 0.064\,\Omega$. With the same assumption, but at 100 MHz, the calculated Q is about 190, very different from the datasheet. Clearly this approximation is of no use above a few megahertz.

For the same component the identical Q factor may be expressed in terms of the equivalent parallel resistance (R_P) in ohms as shown in Figure 4-6(b). In this case it is the ratio of the magnitudes of the reactive *current* (I_L) and the resistive *current (* I_L *)*:

$$Q = \frac{I_L}{I_R} = \frac{R_P}{\omega L} = \frac{R_P}{2\pi fL} \tag{4.8}$$

A similar calculation but using the parallel equivalent circuit (4.8) gives a Q factor at 100 MHz of about 2, nothing like the measured Q factor. No doubt there is a better equivalent circuit for this part which performs well over frequency. That would be a good question for a high frequency inductor designer.

Similar expressions for the C-R series and parallel circuits are also included in Figure 4-6.

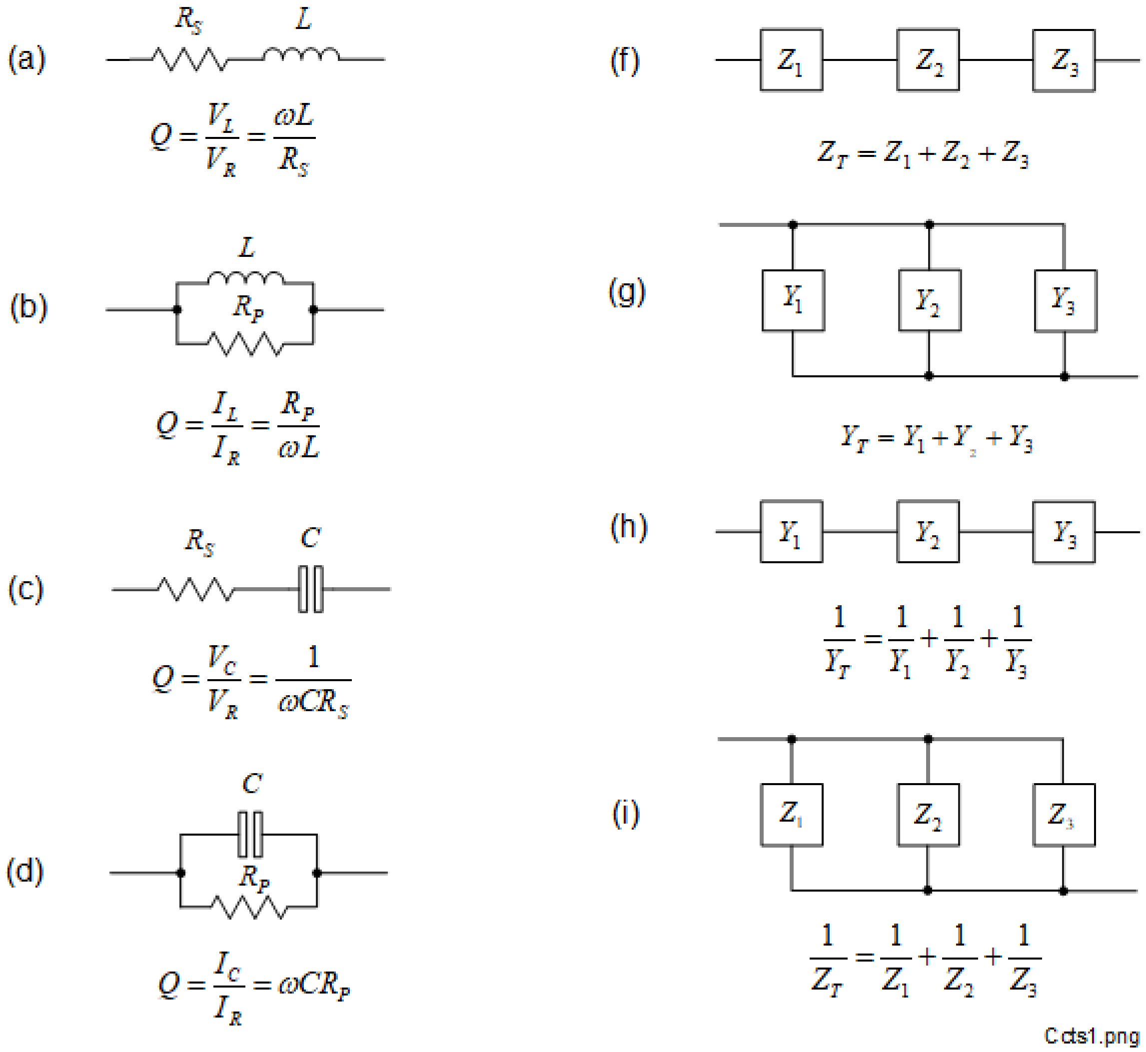


Figure 4-6 Some simple equivalent circuit schematics and equations for various combinations of *ideal* resistors, capacitors and inductors. The Q-factor formulas for the series and parallel equivalent circuits have reciprocal relationships.

Each Smith Chart construction is performed for one frequency. Each of the simple series or parallel equivalent circuits, as shown in Figure 4-6(a) to (d), applies also at one frequency. These are the simplest series or parallel configurations which may be mapped onto the Smith Chart. As discussed in Section 4.3.3, when constructing networks we must determine whether normalized impedance (z) or normalized admittance (y) 'mode' is appropriate and keep track of changes between them. We already know from the schematics for combined series and parallel networks, shown in Figure 4-6(f) to (i), that impedances and admittances provide the simplest complex arithmetic: impedances for series circuits, admittances for parallel circuits. Then complex sums and differences are readily applied to the Smith Chart for conjugate matching. Then, as we saw in the example in Section 3.4, it is a simple graphical translation to convert between normalized impedances and normalized admittances. Using these, when required we can easily convert into and out of their respective un-normalized values: impedance (Z) or an admittance (Y).

4.3.6 AC Networks

An ideal or 'perfect' inductor or capacitor contains no resistance element. However, as we noted in Section 4.3.4.3, at higher frequencies (but still within the limits of lumped element components) we must use the high frequency data provided by the manufacturer or our own reliable measurements. We looked at the examples from Coilcraft in Section 4.3.4.1 in which we could resolve the inductor to either a series or parallel equivalent L-R circuit. Therefore, in the course of our Smith Chart graphical construction, we could use whichever was the more convenient. Practical reactive components may be represented by one of the circuits shown in Figure

4-6(a) to (d) for each measurement frequency, both series and parallel equivalent LR and CR circuits. Provided the frequency is unchanged and the series or parallel resistance has been correctly calculated like the example described in Section 4.3.4.3, we may use either.

4.3.7 General Impedance, Admittance and Related Expressions

Table 4-1 shows a summary of impedance, admittance, reactance and susceptance expressions for ideal resistors, inductors and capacitors, both absolute and normalized. It is useful to read this together with Figure 4-6. To re-iterate, impedance and admittance parameters are used for elements connected in series and parallel respectively. The same argument applies to their normalized versions. The symbols used were defined in Sections 3.4 and 4.2.

Table 4-1 Expressions for absolute and normalized impedances and admittances for ideal resistance, inductance and capacitance elements. These will form parts of the associated complex expressions.

Parameter	Absolute		Normalized	
	Impedance Z	Admittance Y	Impedance z	Admittance y
Resistance R	R	$G = \frac{1}{R}$	$r = \frac{R}{Z_0}$	$g = \frac{1}{RY_0}$
Inductance L	$jX_L = j\omega L$	$-jB_L = -j\frac{1}{\omega L}$	$jx_L = j\frac{\omega L}{Z_0}$	$-jb_L = -j\frac{1}{\omega L Y_0}$
Capacitance C	$-jX_C = -j\frac{1}{\omega C}$	$jB_C = j\omega C$	$-jx_C = -j\frac{1}{\omega C Z_0}$	$jb_C = j\frac{\omega C}{Y_0}$

More information is included below.

- Impedance (Z) and Normalized Impedance (z)

An impedance is the complex number with the real coefficient, resistance (R), imaginary coefficient, reactance (X) and units ohms (Ω). Upper-case and lower-case symbols are used for the un-normalized and normalized versions respectively. A common convention is to subscript the reactance by L for an inductor or C for a capacitor, X_L and X_C respectively. The signs for the complex coefficients are included in Table 4-1: positive for an inductor and negative for a capacitor.

For (absolute) impedance:

$$Z = R \pm jX \quad \Omega \tag{4.9}$$

For normalized impedance:

$$z = \frac{Z}{Z_0} = \frac{R}{Z_0} \pm j\frac{X}{Z_0} = r \pm jx \tag{4.10}$$

Normalized impedance is unitless.

- Admittance (Y) and Normalized Admittance (y)

An admittance is the complex number with the real coefficient conductance (G), imaginary coefficient susceptance (B) and unit siemens (S). Upper-case and lower-case symbols are used for the un-normalized and normalized versions respectively. A common convention is to subscript the susceptance by L for an inductor or C for a capacitor, B_L and B_C respectively. The signs for the complex coefficients are included in Table 4-1: positive for a capacitor and negative for an inductor.

For (absolute) admittance:

$$Y = G \pm jB \quad S \tag{4.11}$$

For normalized admittance:

$$y = \frac{Y}{Y_0} = \frac{G}{Y_0} \pm j\frac{B}{Y_0} = g \pm jb \tag{4.12}$$

- Characteristic Impedance and Characteristic Impedance

The characteristic admittance (Y_0) is the reciprocal of the characteristic impedance (Z_0).

$$Y_0 = \frac{1}{Z_0} \tag{4.13}$$

4.3.8 How Do We Represent S-Parameters on Smith Charts?

The result of a correctly calibrated and measured scattering parameter (S-parameter) measurement made on a device may be plotted on a Smith Chart if the stimulus and response port indices are identical[28]. The S-parameter element must therefore be of the form S_{mm} where m is an integer[29]. S_{mm} will be identical to the reflection coefficient measured at the same port under the same conditions. This follows from the definition of S-Parameters [25]. Also, the characteristic impedance of the measurement must be identical to that used for the Smith Chart.

Other S-parameter measurements may be used elsewhere as required but are not applicable for Smith Chart plotting. S-parameters are derived from linear ratios of voltage vectors so they are unitless[30]. The port must not operate as a source but be a passive load which obeys the conditions required for valid S-parameter measurements.

Following on from the equations for normalized impedance and normalized admittance, (3.15) and (3.16) respectively, we may simply replace the voltage reflection coefficient (ρ) with the calibrated and measured S-parameter at the same frequency (S_{mm}):

$$z = \frac{1+\rho}{1-\rho} = \frac{1+S_{mm}}{1-S_{mm}} \tag{4.14}$$

$$y = \frac{1-\rho}{1+\rho} = \frac{1-S_{mm}}{1+S_{mm}} \tag{4.15}$$

We still have to decide whether we are using lumped element or non-lumped element (distributed element) conditions. As we discussed in Section 4.3.4, this decision is made after considering the shortest wavelength

[28] A properly performed calibration on a vector network analyzer using a calibration kit will correct for total phase and be suitable for distributed or lumped element conditions.
[29] Any index number may be chosen which is appropriate to the system under consideration.
[30] The dB form only applies to the (logarithmic) magnitude. If required, the spatial phase may be shown separately.

which occurs at the highest operating frequency after allowing for any dielectric effects which cause ‘slowing’ of the propagation velocity from that for free space.

5 Examples

This chapter provides some worked examples involving Smith Charts. This article is about Smith Chart graphical methods so we will not achieve the same sort of accuracy using these as we would using a dedicated application written in, for example, Excel or Matlab.

We will refer to points on the Smith Chart as P_n or Q_n in normalized impedance and normalized admittance respectively, where n is an integer.

For all absolute quantities we will use System International (SI) units. These are listed together with the parameter names, units, abbreviations and multiplication factors in Table 1-1 and Table 1-2.

5.1 Example 1: Some Imperfectly Matched Transmission Lines

5.1.1 Question

Using lumped element conditions, represent the circuits listed in Table 5-1 on a Smith Chart using a system impedance of 50 Ω and an operating frequency of 100 MHz. What are the results for: voltage reflection coefficient magnitude, return loss and VSWR?

Table 5-1 Various reactive loads for Smith Chart representations.

Series			**Parallel**		
R (Ω)	**L (H)**	**C (F)**	**R (Ω)**	**L (H)**	**C (F)**
34		1n	45	-	50p
53	160n	-	61	100n	-

5.1.2 Answer

Refer to the equations and circuits shown in Table 4-1 and Figure 4-6 and the Smith Chart constructions shown in Figure 5-1 for the series circuits and Figure 5-2 for the parallel circuits. To process series and parallel circuits, it is usually easier to use impedances and admittances respectively. Starting with the series circuits using the Smith Chart as a (normalized) impedance chart.

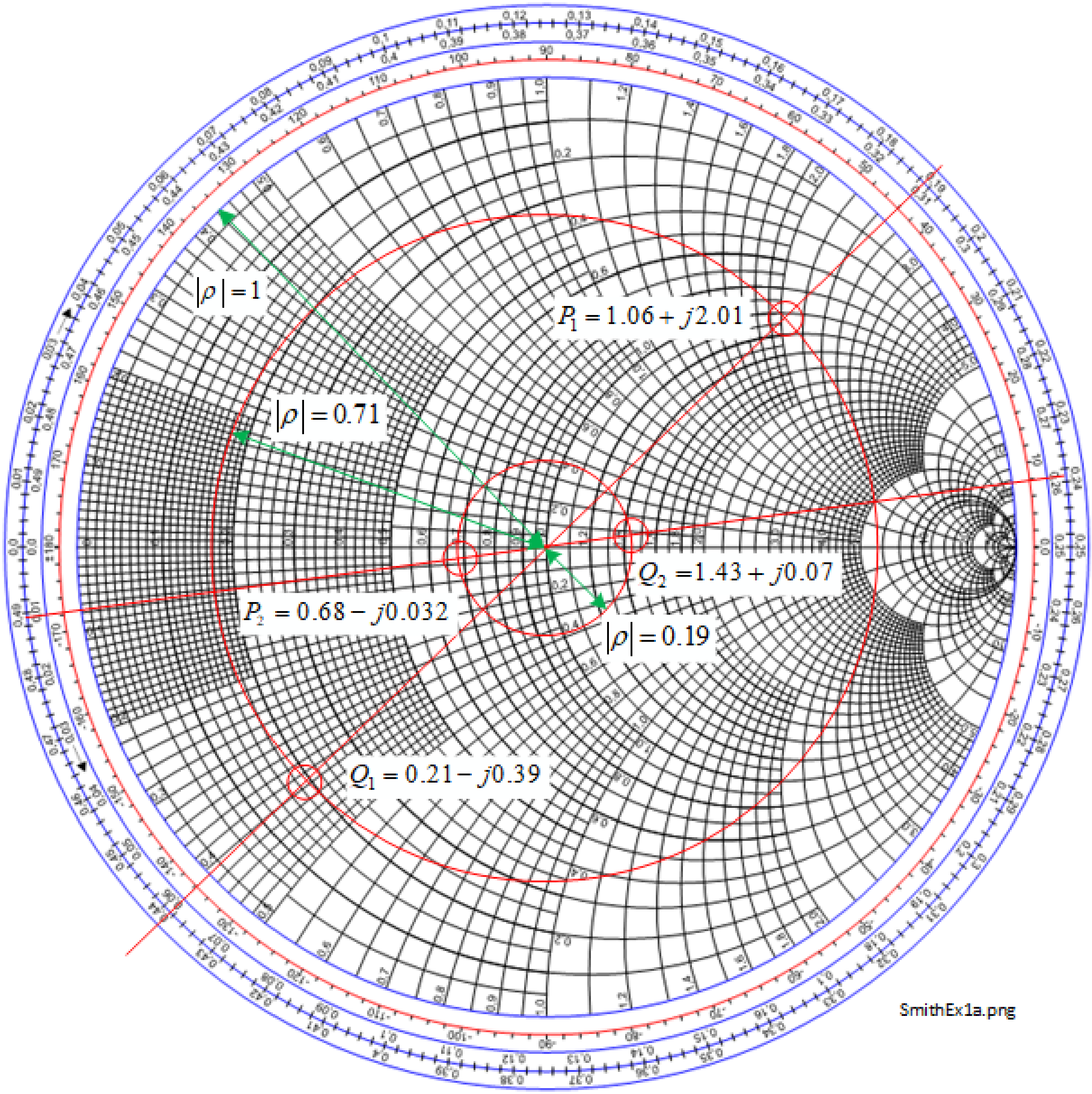


Figure 5-1 Smith Chart construction, Example 1 (a) series RC and series RL circuits, starting at P1 as a normalized impedance chart with Z_0 = 50 Ω.

The 160 nH inductor, part of the series R-L circuit has a reactive magnitude of 2.01 Ω at 100 MHz. Therefore, the impedance Z of this circuit at 100 MHz is:

$$Z = 53 + j100.5 \quad \Omega \tag{5.1}$$

The normalized impedance z is this impedance divided by the chosen system impedance (Z_0) of 50 Ω, so:

$$z = \frac{Z}{Z_0} = 1.06 + j2.01 \tag{5.2}$$

This is plotted at P1 on the Smith Chart and a circle drawn through this point, its center coincident with the center of the Smith Chart ($z = 1 \pm j0$). The impedance has units of ohms and the normalized impedance is unitless. The magnitude of the reflection coefficient is $|\rho| = 0.71$, which is the linearly scaled radius of this circle, the outer circle radius being $|\rho| = 1$. Using (3.39) and (3.40) we may calculate the return loss and

VSWR which are 3.1 dB and 5.6:1 respectively. Alternatively, the voltage reflection coefficient, and therefore its magnitude, may be calculated from (3.15) or (3.16) which are functions of z or y respectively.

Transforming P1 through 180º to Q1 changes the Smith Chart to a (normalized) admittance type, also changing the scaling to normalized admittance. Reading from the scales at Q1:

$$y = 0.21 - j0.39 \qquad (5.3)$$

To convert this back to *absolute admittance* it must be multiplied by the *system admittance*, $Y_0 = 1/Z_0 = 0.02 \quad S$. Therefore:

$$Y = yY_0 = 0.0042 - j0.0078 \quad S \qquad (5.4)$$

Similarly, but for the series RC circuit, following the path from P2 to Q2, the impedance and normalized impedance are given by the following:

$$Z = 34 - j1.592 \quad \Omega \qquad (5.5)$$

$$z = \frac{Z}{Z_0} = 0.68 - j0.032 \qquad (5.6)$$

$$Y = 0.0296 + j0.0016 \quad S \qquad (5.7)$$

$$y = 1.48 + j0.08 \qquad (5.8)$$

The corresponding scalar reflection parameters are $|\rho| = 0.19$, return loss 14.4 dB and VSWR = 1.47:1.

The parallel circuits shown on the right of Table 5-1 use the constructions shown in Figure 5-2. For these we will follow a similar process to that applied to the series circuits, with modifications for admittances instead of impedances.

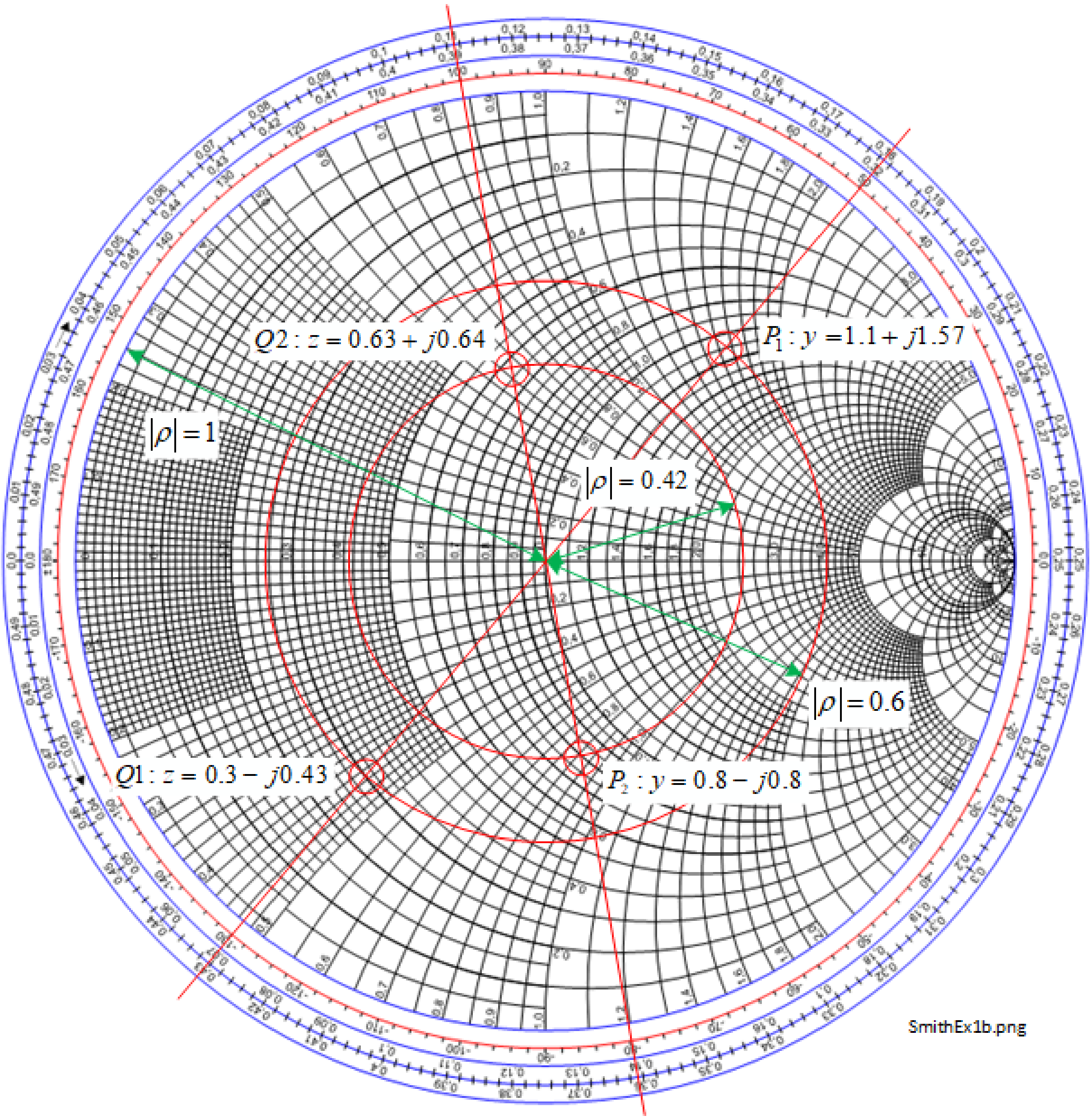


Figure 5-2 Smith Chart construction, Example 1 (b), parallel RC and parallel RL circuits, starting at P1 as a normalized admittance chart.

For the parallel R-C circuit (R = 45 Ω, C = 50 pF), the calculated admittance at 100 MHz is:

$$Y = 0.022 + j0.0314 \quad S \tag{5.9}$$

Dividing this value by Y_0, the normalized admittance is:

$$y = \frac{Y}{Y_0} = 1.1 + j1.57 \tag{5.10}$$

This is plotted at P1. Converting this to normalized impedance by the 180º translation, at Q1 gives:

$$z = 0.3 - j0.43 \tag{5.11}$$

This may be converted back to impedance by multiplying by Z_0:

$$Z = zZ_0 = 15 - j21.5 \quad \Omega \tag{5.12}$$

The reflection coefficient magnitude is $|\rho| = 0.6$. The associated return loss is 4.5 dB and VSWR is 4:1.

For the parallel R-L circuit (R = 61 Ω, L = 100 nH), the calculated admittance at 100 MHz is:

$$Y = 0.017 - j0.016 \quad S \tag{5.13}$$

From this, the normalized admittance plotted at P2 is:

$$y = 0.82 - j0.80 \tag{5.14}$$

Performing the 180º translation to the Q2 normalized impedance is:

$$z = 0.63 + j0.64 \tag{5.15}$$

Multiplying this by Z_0 gives the actual impedance:

$$Z = 31.5 + j32 \quad \Omega \tag{5.16}$$

In this case $|\rho| = 0.42$, return loss = 7.7 dB and VSWR = 1:2.4.

5.2 Example 2

5.2.1 Question

What Are the Limitations of Using Scalar Reflection Parameters Across Frequency?

5.2.2 Answer

The *scalar* reflection parameters are: reflection coefficient magnitude $|\rho|$, return loss and VSWR. Return loss magnitude is a logarithmic measure using the decibel (dB) definition and the others are linear. Probably the most popular of these parameters used in recent years is return loss as this integrates well with the widespread adoption of other dB based logarithmic scales. The Smith Chart in Figure 5-3 shows 3 circles of return loss values: 3 dB, 10 dB and 20 dB. These could have been expressed circles of constant reflection coefficient magnitudes or VSWRs.

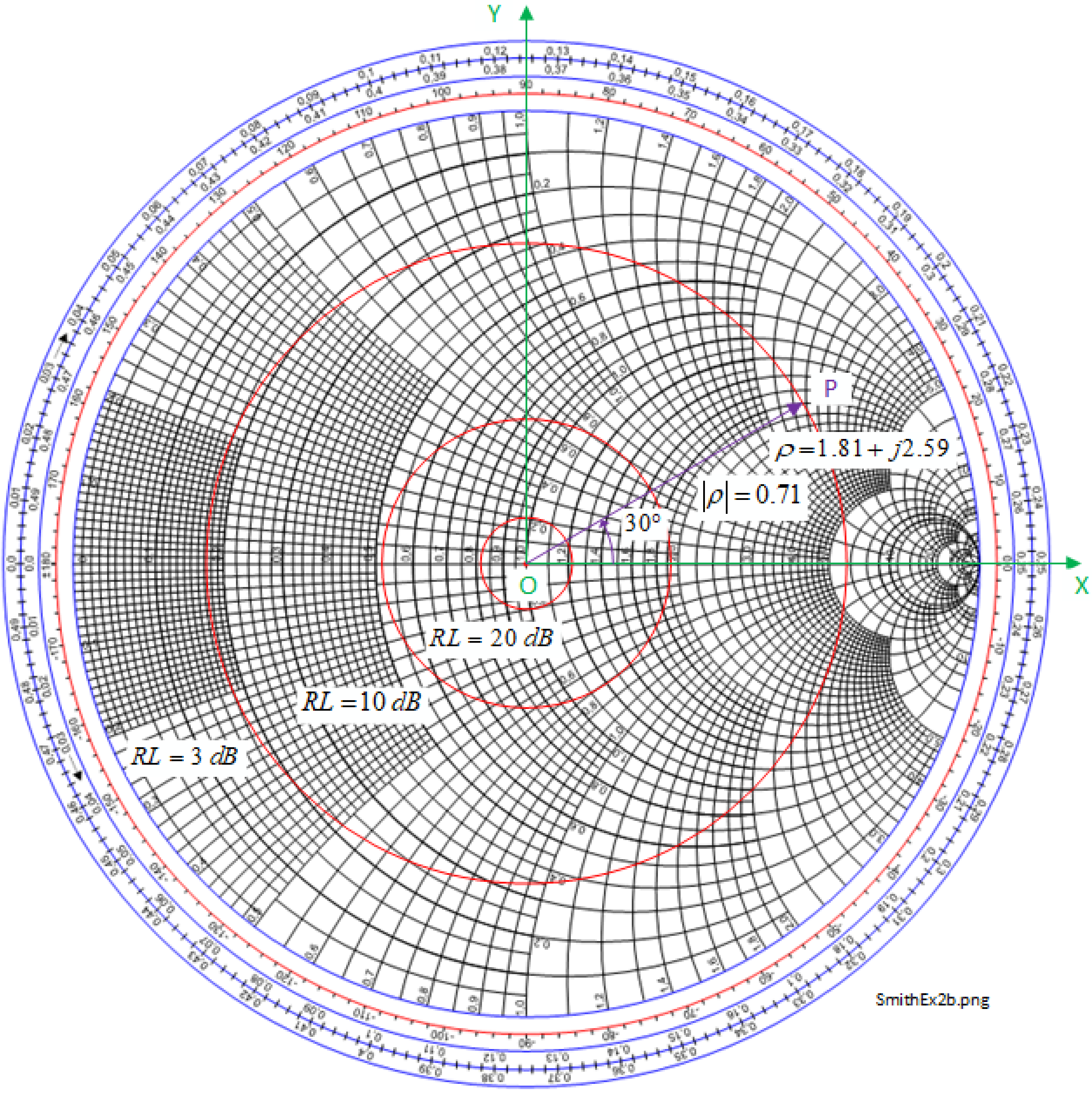


Figure 5-3 Constant magnitude reflection coefficient circles corresponding to return loss values of 3 dB, 10 dB and 20 dB with some vector composition. All values are normalized so a value for Z_0 is not applicable at this stage.

As an example, a reflection coefficient vector OP is shown in purple as one possible value for the 3 dB return loss circle, equivalent to $|\rho| = 0.71$. The orientations of the rectangular x and y axes are OX and OY respectively, with positive angles starting at OX in a counter-clockwise direction. OX starts at the center of the Smith Chart at $z = 1 \pm j0$, positive being in the direction of infinity to the right[31].

The angle of OP is 30º so it could be represented in linear 'magnitude-angle' form as $\rho = 0.71\angle 30°$ or in rectangular form as $z = 1.81 + j2.59$. A logarithmic (dB) scale could be used for $|\rho|$. This is the same magnitude as the return loss value but it has a negative sign. There are an infinite number of ρ values and therefore z values around the constant magnitude circle.

[31] This is the most common Smith Chart orientation for impedance, but could be used for admittance.

Usually a 20 dB return loss ($|\rho| = 0.1$ or a VSWR of 1.22:1) is a good match performance at 100 MHz. Since this is a scalar quantity, there are an infinite range of associated impedance vectors. We will consider a few examples. Assuming that this (imperfect) load comprises a series R-L or R-C circuit, Table 5-2 shows the spread of R, L and C values as the angle of the reflection coefficient moves through 30º steps from 0º to 330º, at 100 MHz and $Z_0 = 50\,\Omega$. A similar table could be constructed for a parallel R-L or R-C circuit.

Table 5-2 An example spread of load circuit elements, all of which produce a return loss of 20 dB at an operating frequency of 100 MHz and a system impedance Z_0 of 50 Ω.

ρ Angle (deg)	Series R-L-C Circuit		
	R (Ω)	L (nH)	C (pF)
0	61.0	0	0
30	59.2	9.5	0
60	54.4	15.2	0
90	49.0	15.8	0
120	44.6	12.4	0
150	41.8	6.7	0
180	41.0	0	0
210	41.8	0	377
240	44.6	0	204
270	49.0	0	161
300	54.4	0	167
330	59.2	0	266

A scalar return loss measurement does not provide an indication of the actual (vector) impedance. For the case of 20 dB return loss at 100 MHz, all we can say is that a range of impedances are possible, of which a few examples are shown in Table 5-2.

However, a return loss measurement is usually quicker, requires less skill and less expensive equipment that a VNA measurement so it is suited to high confidence measurements. All of these factors must be taken into account when specifying a return loss specification.

For different frequencies, similar results could be calculated but the reactive components would differ. For example, if the operating frequency was 1 GHz instead 100 MHz, the capacitor and inductor values would be reduced by 10, so an even higher return loss may be appropriate at this frequency.

5.3 Example 3

5.3.1 Question

Design a low loss passive matching network between a 50 Ω source and a 75 Ω load for operation at 100 MHz.

5.3.2 Answer

The schematic to represent the requirement is shown in Figure 5-4. NET1 contains the passive network that is required. It has 2 ports, designated 1 and 2, represented by the circled numbers. Ports 1 and 2 have nominal impedances of 50 Ω and 75 Ω respectively. The top diagram shows a Thevenin equivalent source of 50 Ω feeding a load of 75 Ω and the bottom diagram shows another Thevenin equivalent source of 75 Ω feeding a load of 75 Ω. This demonstrates that both the source and the load must be *simultaneously* matched, which ever port is used for the source and load, provided of course they are the correct impedances. We could not, for example use a single 25 Ω series resistor as this would match the 75 Ω port but the 50 Ω port would see 100 Ω.

As the network is passive, but not resistive, the bandwidth is narrow and it is required to be low loss so it may be designed with reactive parts: capacitors and/or inductors. One option might be to use a transformer. Provided one was designed with the correct turns ratio, of suitable gauge wire and the core, perhaps a low-loss ferrite material at this frequency, it could provide a good broadband solution[32]. However, that is a lot of work and a broad band is not required. Another option might be to use resistors only. This would give a good wideband match but be very lossy[33].

The following example is just one possible method to demonstrate the principles involved, other matching configurations are available such as those described in Section 4.3.3.

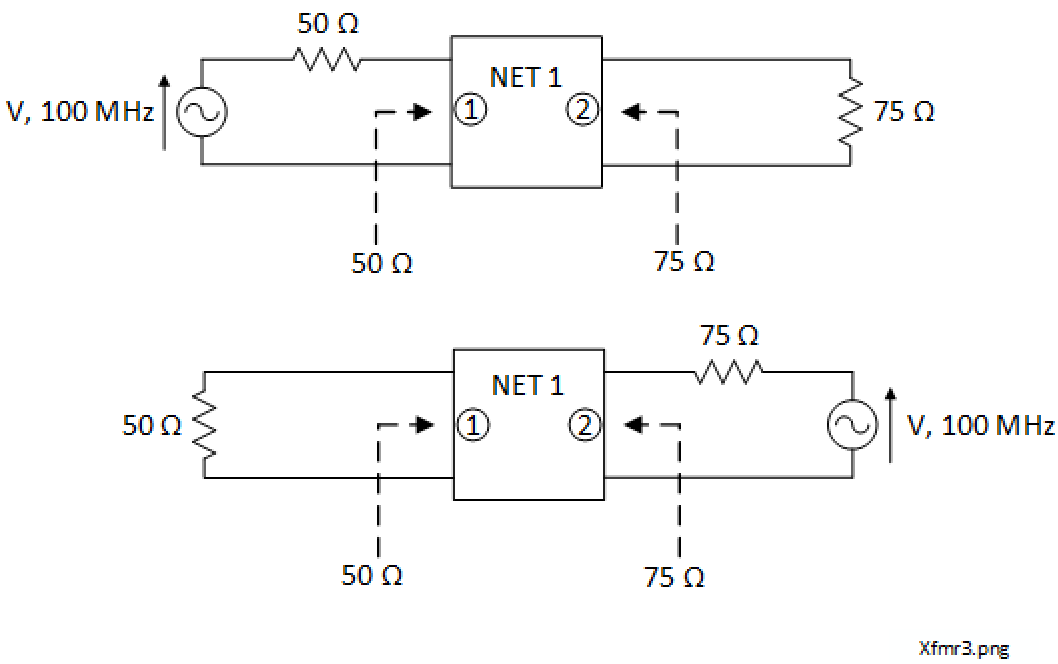


Figure 5-4 Example 3: A schematic of the requirement for the matching network, NET1. The correct impedances, close to 50 Ω or 75 Ω at 100 MHz, must be seen from both directions *simultaneously*.

As NET1 is passive unlike, for example an amplifier, it is considered to be bilaterial: signals in both directions must the considered. Using the Thevenin equivalent constant voltage source, the generator symbol may be replaced with a short circuit to determine the impedance seen when it is replaced by a load.

The circuit schematics for the Smith Chart steps are shown in Figure 5-5. The characteristic impedances used were Z_0 = 50 Ω and Z_0 = 75 Ω for the respective 50 Ω and 75 Ω inputs.

5.3.2.1 Conjugate Matching Method

Smith Chart matching uses conjugate matching to impedances for series connected elements, and to admittances for parallel connected elements. Conversions to and from (un-normalized) values are applied as required, using either the characteristic impedance (Z_0) or the characteristic admittance (Y_0). All of the formulas including signs of the complex coefficients are summarized in Table 4-1

[32] Another option might be a 'transmission line transformer' using ferrite cores.
[33] Simple resistive minimum loss matching networks are cheaper than transformer types and adequate for many purposes but excluding low loss.

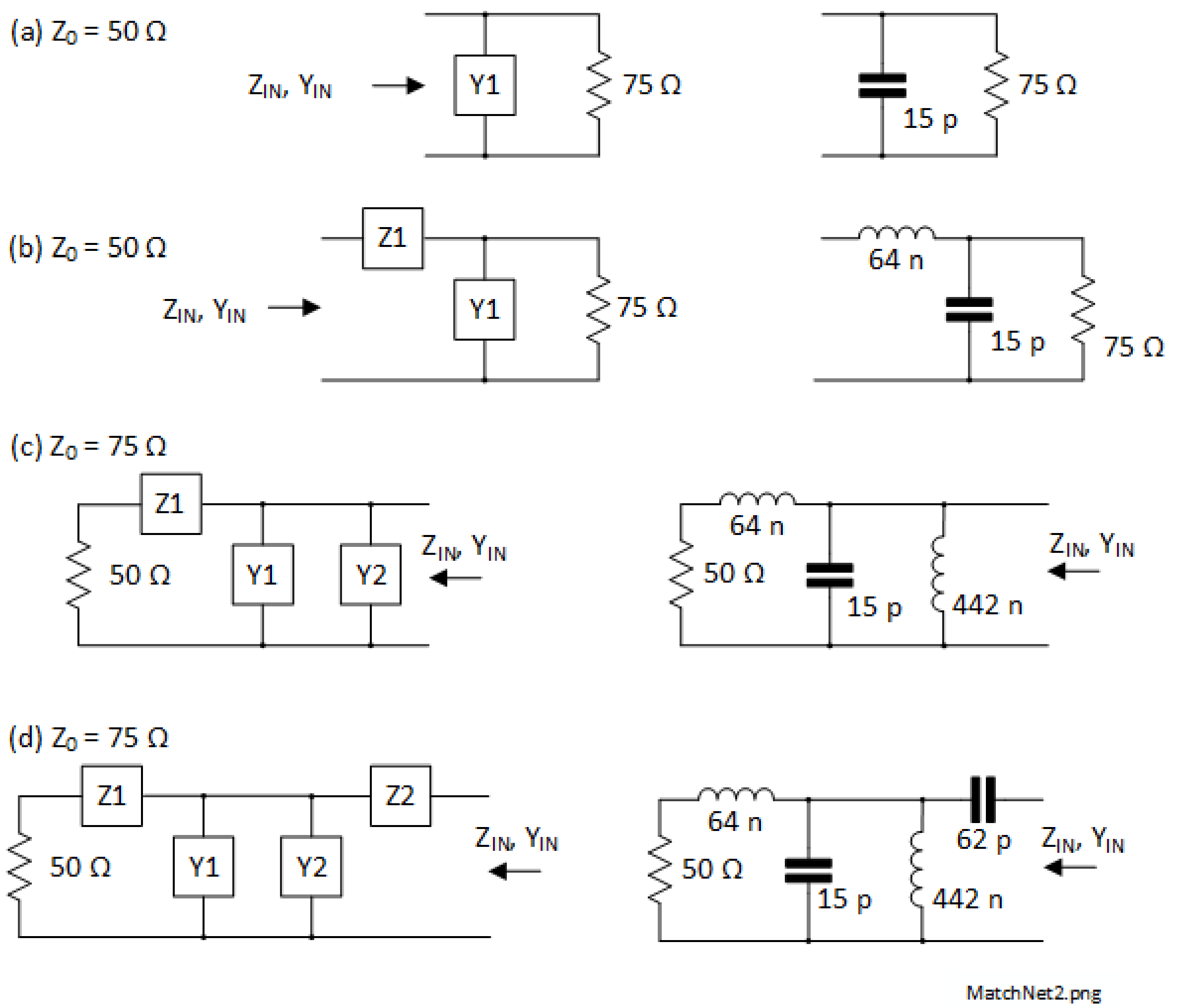


Figure 5-5 Example 3: the Smith Chart steps required to develop the matching network looking into the 50 Ω port then looking into the 75 Ω port. Values have been rounded to the nearest pF and nH. Only one iteration was necessary to get good performance shown by the S-parameter simulation results.

The Smith Chart constructions are shown in Figure 5-6, Figure 5-7 and Figure 5-8. The impedances used are 50 Ω, 75 Ω and 75 Ω respectively.

The following method was used:

- The network was initially matched from 75 Ω to 50 Ω. Then the Thevenin source was replaced by a short circuit, retaining the 50 Ω source resistance, and the circuit analyzed from the 75 Ω end using Matlab® RF Toolbox [5] [6].
- The match for 75 Ω was applied.
- The matches at both ends were measured using Matlab® RF Toolbox.
- The transmission performance across a wider band was estimated by connecting 2 identical networks back to back at the 75 Ω ports, then measuring the (now 50 Ω) transmission S-parameters (S_{21}, S_{12}). The transmission was estimated at one half of this value in dB.
- Points Pn and Qn represent values in the normalized impedance (z) and normalized admittance (y) planes respectively, using the appropriate Z_0 value.
- z to y and y to z transformations were applied as required between the Pn and Qn points.
- Conjugate matching was applied using z for series elements and y for parallel elements.

- Element values were calculated using the information shown in Table 4-1 at the operating frequency of 100 MHz.

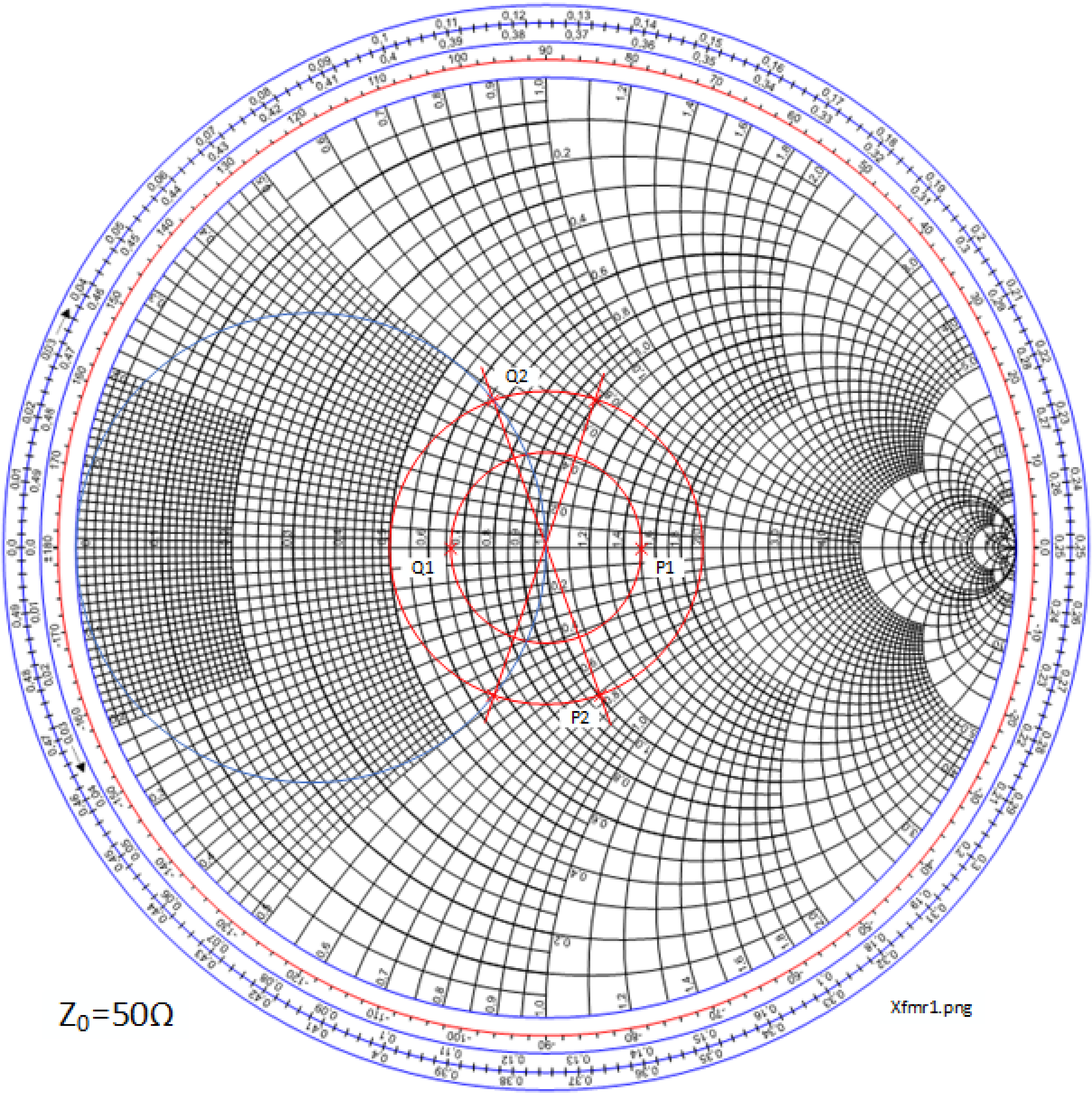


Figure 5-6 Smith Chart construction for Example 3 (a) at 100 MHz. Components looking into the 50 Ω port, 75 Ω load: P1 ($z = 1.5 \pm j0$), Q1 ($y = 0.67 \pm j0$). Q1 to Q2 ($+j0.47$): parallel capacitor 15 pF. Q2 to P2: return to z. P2 to $z = 1 \pm j0$: $+j0.8$ series inductor 64 nH.

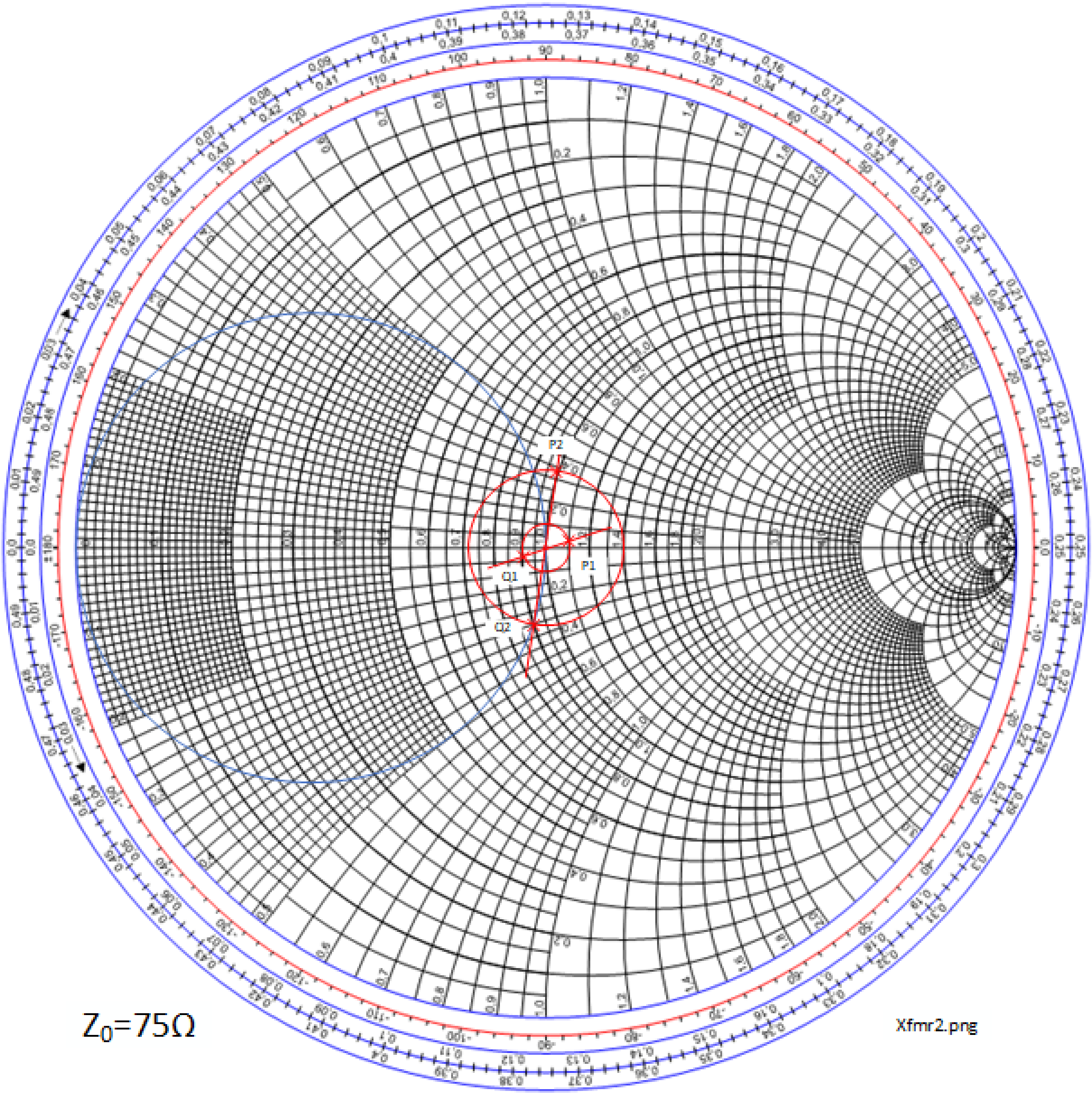


Figure 5-7 Smith Chart construction for Example 3 (b). Components looking into the 75 Ω port. For the detailed steps, refer to the expanded center area shown in Figure 5-7.

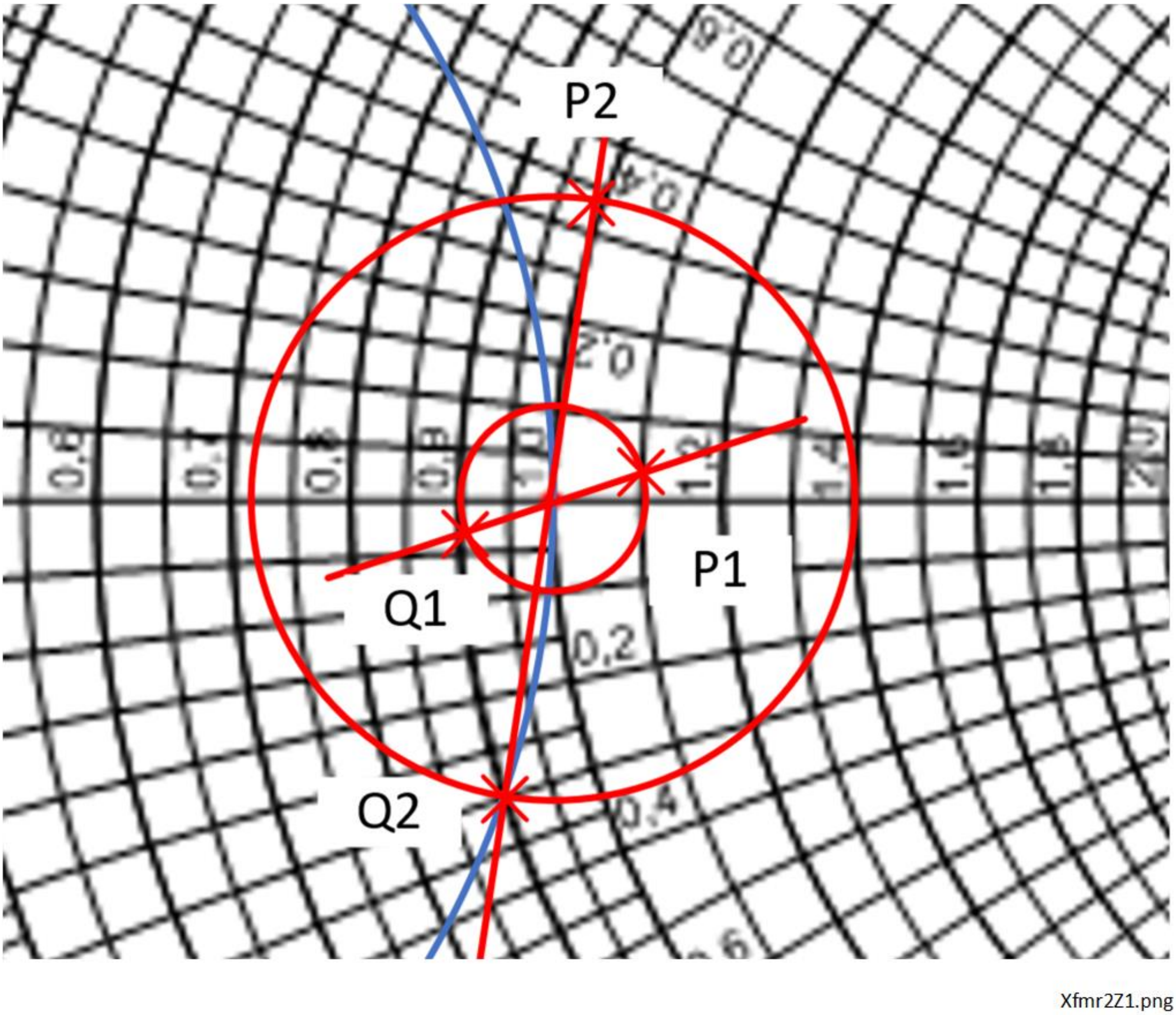


Figure 5-8 Example 3 (c): Smith Chart expanded area for matching to 75 Ω. Further to a Matlab® RF Toolbox analysis, described in Section 5.3.2.2, P1 ($z = 1.1 + j0.036$) is the uncorrected normalized impedance (Z_0 = 75 Ω), converted to normalized admittance Q1 ($y = 0.9 - j0.03$) using the inner red circle. A parallel 442 nH inductor was used from Q1 to Q2 (on the $y = 1 \pm jx$ blue circle). Q2 to P2 was a y to z conversion to the $z = 1 \pm jx$ circle. P2 to was the final match to $z = 1 \pm j0$ requiring a 62 pF series capacitor.

5.3.2.2 Performance Verification

The Matlab® math application, including the RF Toolbox add-on, was used to measure the actual impedance of the nominally 75 Ω port after the first iteration (75 Ω to 50 Ω) [5] [6]. It was found to be very close to 75 Ω at $z = 1.1 + j0.036$ which is shown at P1 in Figure 5-8, remembering that Z_0 = 75 Ω in this case. This was used as the starting point for matching from 50 Ω to 75 Ω also shown in Figure 5-8. This may have been expected to distort the original match (75 Ω to 50 Ω), but in fact, after some further verification, both matches and the estimated transmission over the wider bandwidth of 50 MHz to 150 MHz were satisfactory.

The circuit for the final result is shown in Figure 5-9, together with the steps taken for verification.

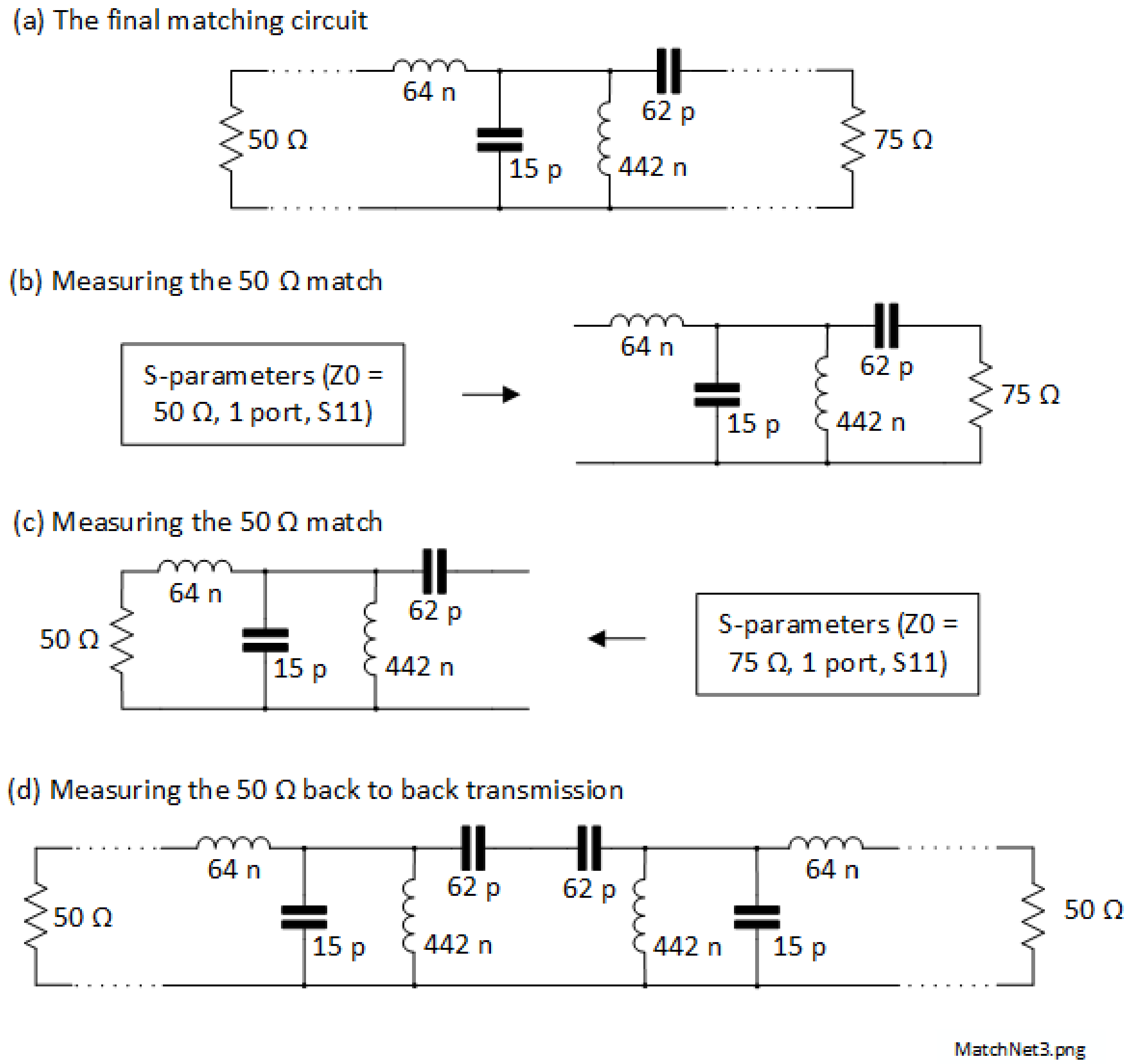


Figure 5-9 Schematic steps for performance verification of the matching circuit.

The results from the verification tests in Figure 5-9(b), (c) and (d) are shown in Figure 5-10.

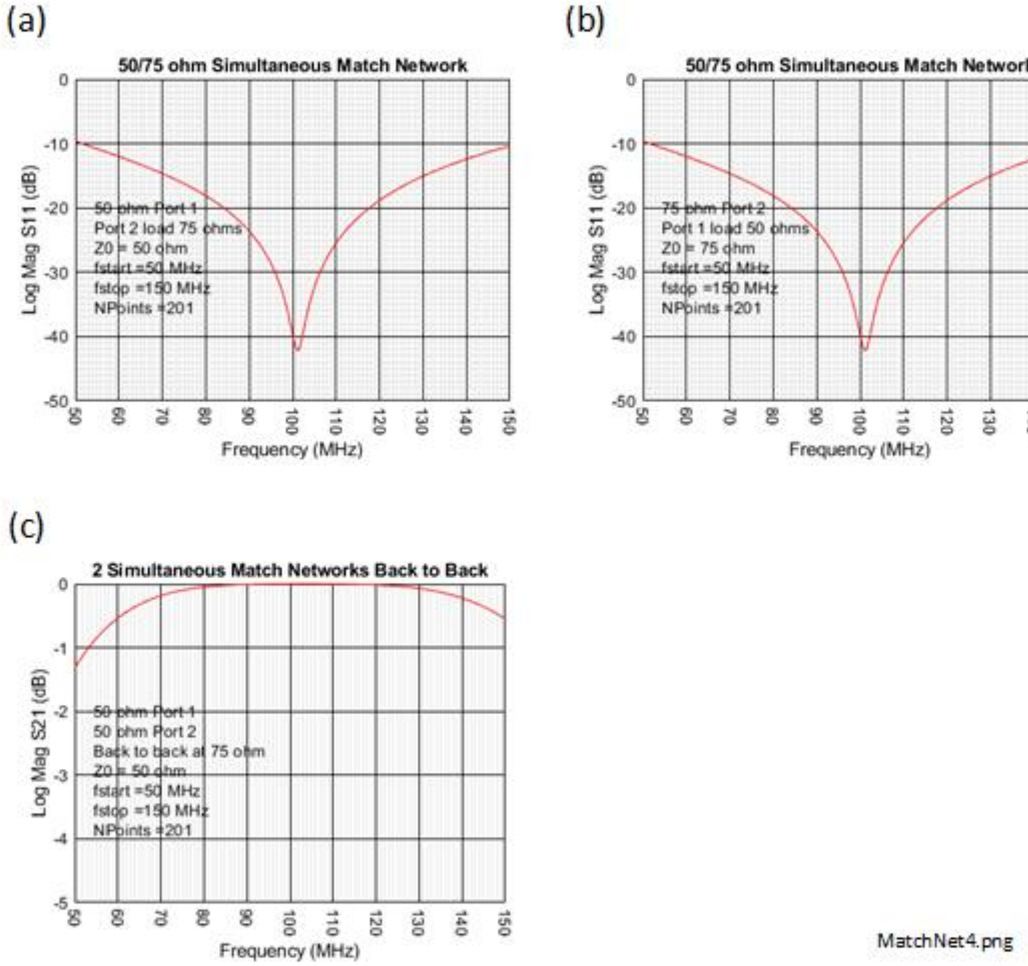


Figure 5-10 S-parameter measurements: (a) 50 Ω port reflection, (b) 75 Ω port reflection, (c) transmission for 2 networks back to back.

5.4 Example 4

5.4.1 Question

A load comprising a 65 Ω resistor and 33 nH inductor in series is connected to the end of a loss free cable of length 330 m. Calculate the return loss of the load for Z_0 = 75 Ω at 100 MHz. Using 75 Ω and 50 Ω Smith Charts, design a circuit to match the other end of the cable to 50 Ω, operating at 100 MHz. The cable is Belden® part MRG5902, characteristic impedance 75 Ω, velocity factor of 66% and loss of 0.12 dB/m at 100 MHz [18].

5.4.2 Answer

In order to determine the impedance change along the cable, we will use the outer (wavelength) circumferential scaling on the Smith Chart. The cable (characteristic) impedance is Z_0 = 75 Ω, so this must also be used for the Smith Chart. With the help of Table 4-1, the normalized load impedance, say at position B, is:

$$z_B = \frac{65}{Z_0} + j\frac{\omega L}{Z_0} = 0.87 + j0.28 \qquad (5.17)$$

This is plotted on the Smith Chart shown in Figure 5-11 at point P1. Then a circle is drawn through this point, centered at $z = 1 \pm j0$, to represent the magnitude of reflection coefficient for z_B, say $|\rho_B|$. This may be confirmed by expressing (3.15) in terms of ρ_B and then calculating its magnitude, remembering that this is for a $Z_0 = 75\ \Omega$ system:

$$|\rho_B| = \left|\frac{z_B - 1}{z_B + 1}\right| = 0.16 \tag{5.18}$$

The return loss RL is defined (3.39) as:

$$RL = -20\log_{10}|\rho_B| = 16\ dB \tag{5.19}$$

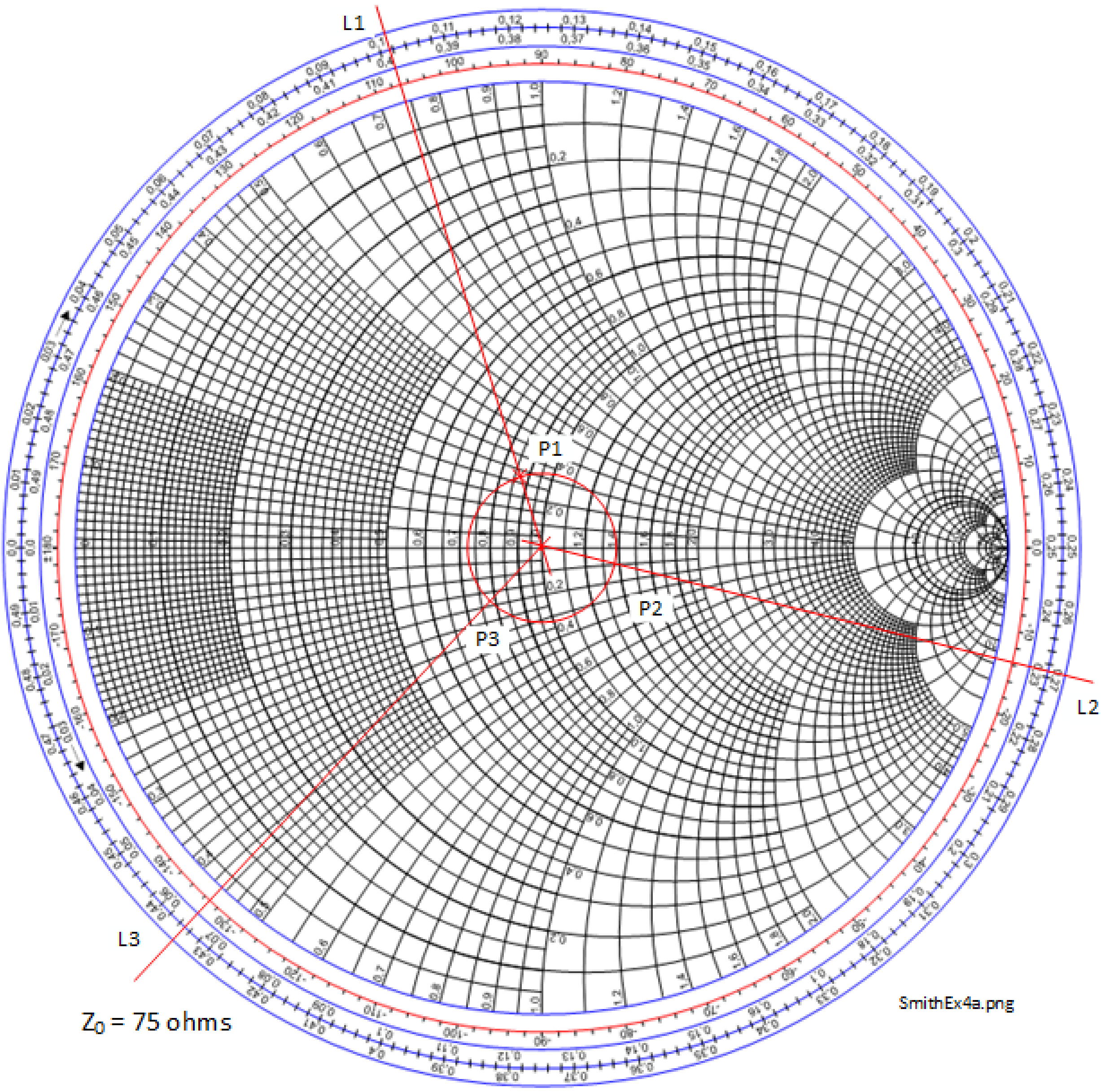


Figure 5-11 Example 4: Smith Chart construction (1) applied to the impedance change along the Z0 = 75 Ω cable.

A straight line is then extended from the center of the Smith Chart, through P1 to the outer scale where it intersects at L1 at 0.102 wavelengths.

To calculate the number of wavelengths at 100 MHz contained within the cable we require the velocity of electromagnetic radiation within the cable after it has been ‘slowed down’ by the dielectric material(s). This is supplied by the cable manufacturer as the velocity factor as 66%, the percentage of the free space value (c). As a decimal fraction we will define it as $VF = 0.66$. Therefore, using (4.6), the wavelength within the cable is given by:

$$\lambda_m = \frac{v}{f} = \frac{VF * v}{f} = 1.98\ m \tag{5.20}$$

The number of wavelengths within the specified cable length of 330 mm (0.33 m) is therefore $0.33/1.98 = 0.167$. To find the normalized impedance at the cable input we must move around the outer scale of the Smith Chart clockwise by 0.167 wavelengths. This is shown in Figure 5-11 from 0.102 to 0.269 (since $0.102 + 0.167 = 0.269$). The normalized impedance (z_A) read from the Smith Chart at P2 is:

$$z_A = 1.37 - j0.11 \tag{5.21}$$

We have to eventually match the input impedance to 50 Ω. There are alternative methods but we will move to a Z_0 = 50 Ω Smith Chart to avoid cluttering this chart excessively. De-normalizing z_A gives us:

$$\begin{aligned} Z_A &= z_A Z_0 \\ &= 1.37 \times 75 - j0.11 \times 75 \\ &= 102.8 - j8.25 \quad \Omega \end{aligned} \tag{5.22}$$

Now we transfer to a Z_0 = 50 Ω Smith Chart which is shown in Figure 5-12. Supposing the normalized impedance on this is z_C, then:

$$z_C = \frac{Z_A}{Z_0} = \frac{102.8}{50} - j\frac{8.25}{50} = 2.056 - j0.165 \tag{5.23}$$

This is plotted at P1 on Figure 5-12.

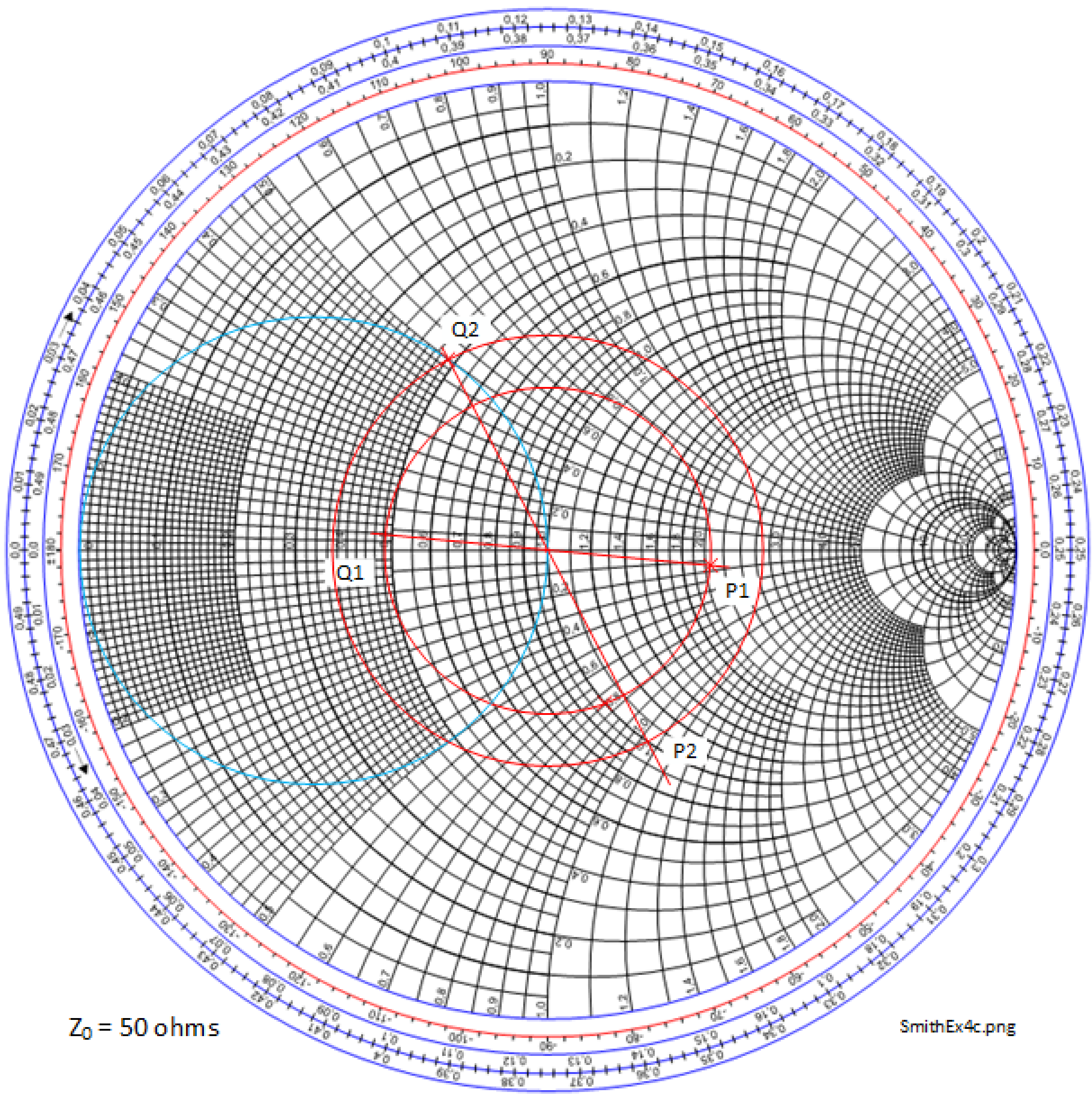


Figure 5-12 Example 4: Smith Chart construction (2) applied to the matching of the input to the 75 Ω cable to 50 Ω.

P1 is then converted to a normalized admittance Q1 ($y = 0.48 + j0.04$) by rotation through 180˚. Q1 is then moved to Q2 ($y = 0.48 + j0.5$) by adding a normalized admittance of $+j0.46$. Q2 is on the constant conductance circle passing through $y = 1 \pm jx$. Referring to Table 4-1, this requires a parallel capacitance C, where:

$$j0.46 = \frac{j\omega C}{Y_0} = j\omega C Z_0 \qquad (5.24)$$

Therefore $C = 14.6\ pF$.

Q2 converts to a normalized impedance at P2 on the $z = 1 \pm jx$ circle at $z = 1 - j1.04$. Adding a normalized impedance (inductance) of $+j1.04$ conjugately matches this to 50 Ω at the center. By referring again to Table 4-1, for the normalized impedance for an inductor:

$$j1.04 = \frac{j\omega L}{Z_0} = \frac{j2\pi fL}{Z_0}$$
$$L = \frac{1.04Z_0}{2\pi f} = 82.7\ nH \quad (5.25)$$

Therefore, a series inductor of 83 nH is required. The final circuit is shown schematically in Figure 5-13.

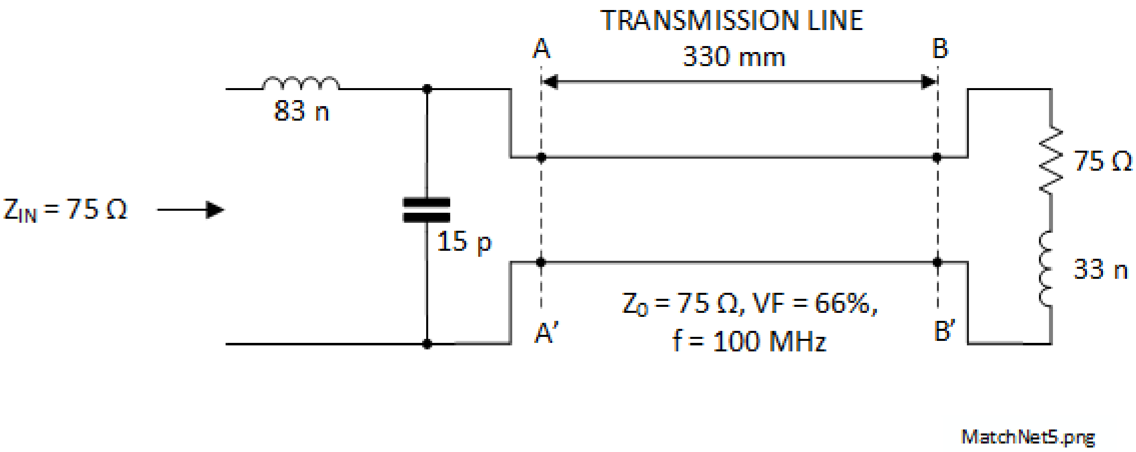


Figure 5-13 Circuit schematic for the result of Example 4. Capacitor and inductor values were rounded off to 1 pF and 1 nH.

5.5 Example 5

5.5.1 Question

Repeat Example 4 in Section 5.4 using only a Z_0 = 75 Ω Smith Chart.

5.5.2 Answer

In Example 4, we initially used the Z_0 = 75 Ω Smith Chart because the impedance of the cable was 75 Ω. Then, to match the cable input to 50 Ω, we used a Z_0 = 50 Ω Smith Chart, ending up at the center ($z = 1 \pm j0$). Alternatively, we could have stayed with the first Smith Chart and matched instead to the 50 Ω position on the 75 Ω Smith Chart.

The first part of the answer is the same as shown on the Smith Chart in Figure 5-11, up to normalized impedance position P2. Now refer to the Smith Chart in Figure 5-14 for the remaining steps.

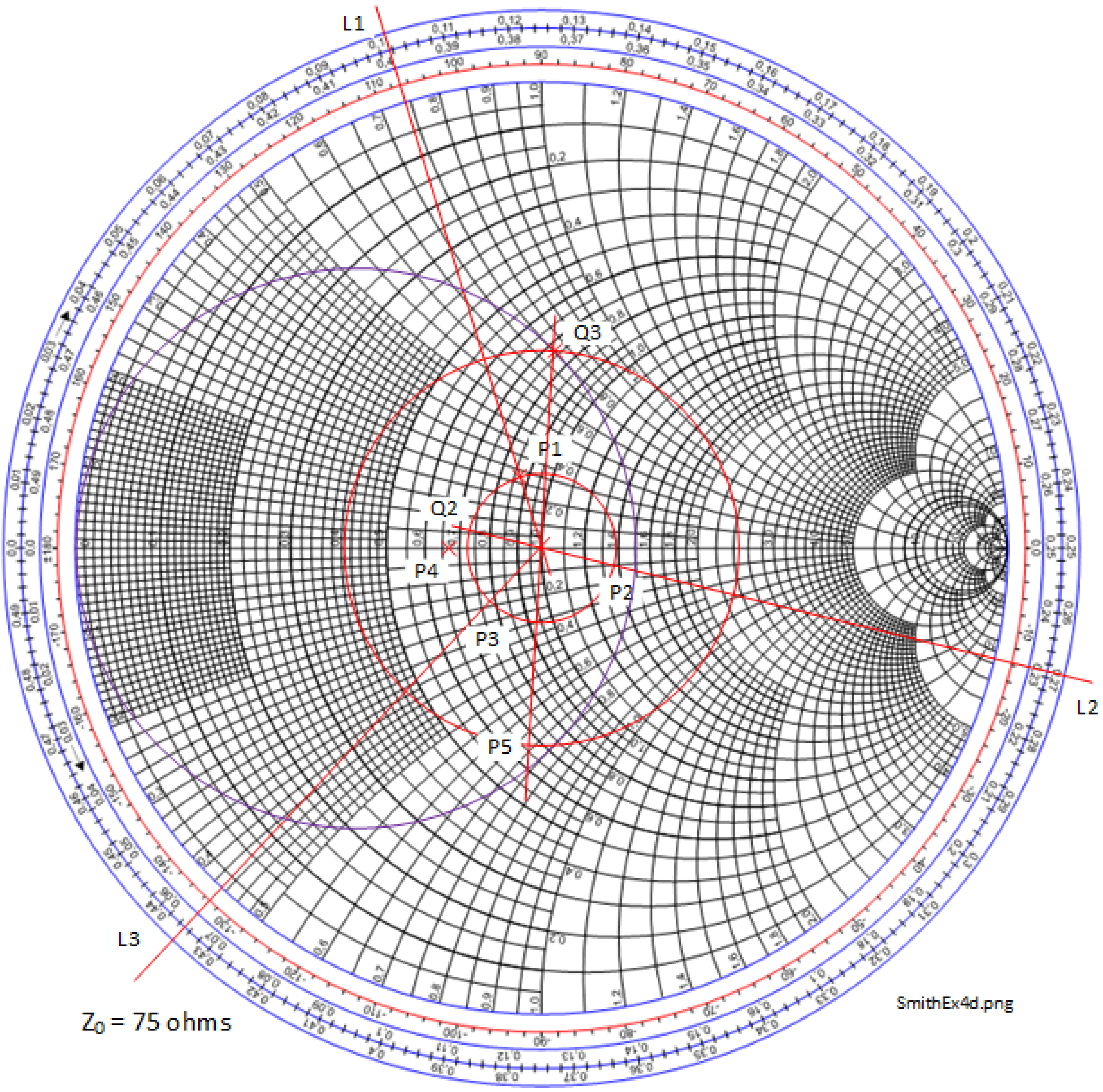


Figure 5-14 Example 5: All calculations are performed using this Z0 = 75 Ω Smith Chart.

In Figure 5-14, the normalized impedance at P2 is that looking into the terminated cable. We wish to eventually move to the point on the Smith Chart which represents the perfectly resistive 50 Ω load (P4). The characteristic impedance in use is Z_0 = 75 Ω. The normalized impedance at P4 is therefore:

$$z = \frac{Z}{Z_0} = \frac{50}{75} = 0.67 \qquad (5.26)$$

P2 is converted to normalized admittance Q2 using the 180º rotation method described in Section 3.4 ($y = 0.73 + j0.05$). The purple circle touching the left side of the Smith chart has been added to assist conjugate matching when normalized impedances and normalized admittances are being used simultaneously on the same chart. Q3 is both on the same constant conductance circle as Q2 and on the purple circle at $y = 0.73 + j0.8$. To move from Q2 to Q3 therefore requires adding a normalized admittance of $j0.75$ which is a parallel capacitance, say C, where:

$$j0.75 = j\frac{2\pi fC}{Y_0} = j2\pi fCZ_0 \quad (5.27)$$

Therefore:

$$C = \frac{0.75}{2\pi fZ_0}F = 15.9\,pF \quad (5.28)$$

A parallel capacitance of 15.9 pF must be added. Q3 is then converted to normalized impedance at P5 again using the 180º method ($z = 0.67 - j0.68$). By virtue of the purple circle described earlier, P5 is already on the constant normalized resistance circle which passes through the final point P4. To move P5 to P4 requires adding a normalized impedance of $j0.68$ which is an inductor, say L, therefore:

$$j0.68 = j\frac{2\pi fL}{Z_0} \quad (5.29)$$

From which

$$L = \frac{0.68Z_0}{2\pi f} = 81.2\,nH \quad (5.30)$$

A series inductor of 81.2 nH is therefore required.

In this example, we have used essentially the same steps of parallel (admittance) and series (impedance) that we did in Example 4 and the results, shown in Figure 5-13, are in good agreement.

6 References

[1] Smith, P. H.; Transmission Line Calculator; Electronics, Vol. 12, No. 1, pp 29-31, January 1931.
[2] Lee, Tim; The Smith Chart Comes Home (President's Column); IEEE Microwave Magazine, November 2015, pp 10-2; IEEE Microwave Theory and Technology Society (MTTS); Institution of Electrical and Electronics Engineers (https://mtt.org).
[3] Coilcraft Square Air Cored Inductor P/N 0806SQ-19N datasheet; https://www.coilcraft.com.
[4] Johanson Technology High-Q 100 pF capacitor datasheet P/N QSCT 0805; https://www.johansontechnology.com.
[5] The Mathworks Inc.; RF Toolbox User's Guide, Matlab R2021a.
[6] The Mathworks Inc ; RF Toolbox Reference, R2022a.
[7] Ramo, Whinnery and Van Duzer (1965); "Fields and Waves in Communications Electronics"; John Wiley & Sons; *The Smith Transmission Line Chart*; pp 35-39.
[8] Pozar, David M. (2005); Microwave Engineering, Third Edition (Intl. Ed.); John Wiley & Sons, Inc.; *The Smith Chart (best known graphical aid)*; pp 64-71. ISBN 0-471-44878-8.
[9] Pozar (op. cit.); *The Terminated Lossless Transmission Line*; pp 57-60.
[10] Gonzalez, Guillermo (1997); Microwave Transistor Amplifiers Analysis and Design, Second Edition; Prentice Hall NJ; *The Smith Chart;* pp 93-103, p154. ISBN 0-13-254335-4.
[11] Hayt, William H Jr.; "Engineering Electromagnetics" Fourth Ed;McGraw-Hill International Book Company; *Transmission Line Equations*; pp 428 433. IBSN 0-07-027395-2.
[12] Davidson, C. W.;"Transmission Lines for Communications with CAD Programs"; Macmillan; *The Smith Chart*; pp 80-85. ISBN 0-333-47398-1
[13] Gonzalez, Guillermo (1997) (op. cit); *Matching Networks*; pp 112-125.
[14] Gonzalez, Guillermo (1997) (op. cit); *Compressed Smith Chart (negative resistance);* pp 98-100.
[15] Gonzalez, Guillermo (1997) (op. cit); *Smith Chart (return loss and S11);* pp109-110.
[16] Morton, A. H., Advanced Electrical Engineering; Pitman Publishing Inc.; *The Impedance Diagram (Smith Chart)*; pp 227-234. ISBN 0 273 401726.

[17] Morton, A. H. (op. cit.); *Transmission Lines*; pp 203-227.
[18] Belden® product P/N MRG5902, coaxial cable, 75 Ω; https://catalog.belden.com/techdata/EN/MRG5902_techdata.pdf.
[19] Pozar (op. cit.); *Multiple Reflections and the Quarter Wave Transformer*; pp 73-77.
[20] Kreyszig, Erwin; Advanced Engineering Mathematics; *Euler's Equations*; pp 589-591; John Wiley & Sons Inc.
[21] Hall, Stephen H,. Heck, Howard L.; High Speed Digital Designs; John Wiley & Sons Inc.; *Ideal Transmission Line Properties*; pp 82-90. ISBN 978-0-470-19235-1.
[22] Hall, Stephen H,. Heck, Howard L. (op. cit); *Transmission Line Reflection and Transmission, Multiple Reflections* ; pp 113-118.
[23] Hall, Stephen H,. Heck, Howard L. (op. cit); *Differential Signalling, Removal of Common Mode Noise*; pp 297-300.
[24] Pozar (op. cit.); *Matching with Lumped Elements (L Networks)*; pp 223-227.
[25] Gonzalez, Guillermo (1997) (op. cit); *S-Parameter Definition;* pp 22-25.
[26] International System of Units (Wikipedia); https://en.wikipedia.org/wiki/International_System_of_Units
[27] Pozar (op. cit.); *Small Reflections and Multi-Section Transformers*; pp 244-255.
[28] Pozar (op. cit.); *Quarter Wave Transformer*; pp 240-244.
[29] Pozar (op. cit.); Resonators and Coupling; pp 291-296.