HIGH SPEED TRANSMISSION LINES

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Summary

This paper looks at the development of various transmission lines starting with DC and discusses their behaviour as the operating frequency is increased. The particular issues of high speed signalling from a balanced source excited differentially, using a balanced transmission line and balanced load are studied.

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1. BACKGROUND

1.1 Radio Frequency Transmission Lines

Radio frequency (RF) transmission lines (TLs) are designed to carry RF power from one location to another with the following properties [5] [6] [1]:

- minimal loss and distortion of the transmitted signal;
- minimal emissions of radiation and susceptibility to external radiation;
- sufficient power handling capability;
- convenient interfacing to equipment and connectors.

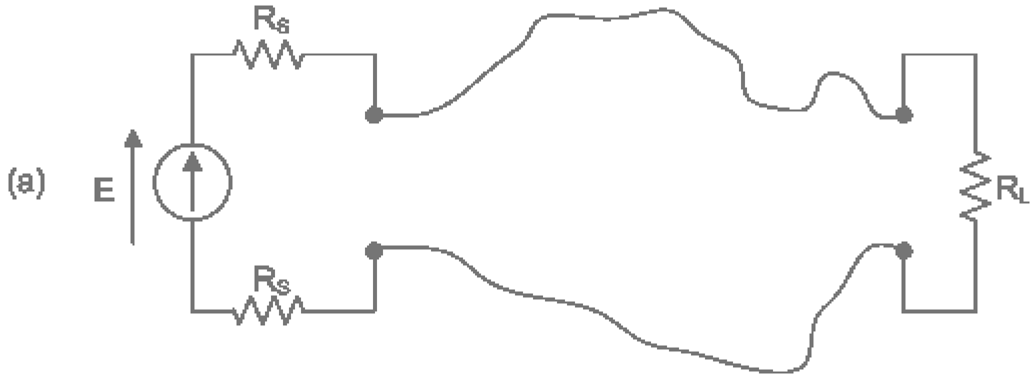
We will now look at their various types, physical constructions and electrical behaviours.

1.2 Let Us Begin at DC

At direct current (DC) or ‘zero’ frequency, it is straightforward to transfer power from a source to a load using a simple circuit as shown in Figure 1-1(a). This has a Thevenin constant voltage (balanced) source on the left feeding a balanced load on the right. The upper and lower wires carry the forward and return currents respectively. Provided the wires between the source and the load have sufficiently small resistance, the load can be located some distance from the source and there is no particular concern about how the wires are routed.

If, instead of DC, we consider the source to be AC with progressively higher frequencies, the reactive properties of the line (inductance and capacitance) become increasingly critical. Figure 1-1(b) is a schematic of the same DC circuit considered in Figure 1-1(a) but for an AC source. Figure 1-1 A simple DC (a) and AC (b) circuit to transfer power from the source on the left to the load on the right. The reactive components become more influential in (b) as the frequency is increased.A typical conductor will inherently have elements of series (self) inductance as shown[2]. There may also be elements of mutual inductance (not shown). Furthermore, the inductive element values will generally increase with frequency, one of the consequences of the skin effect[3]. There may also be elements of capacitance between the lines depending on their relative proximity, larger values for closer spacing[4].

[1] Here we are only studying RF transmission lines. There are many other types of *electrical* transmission lines, for example power transmission lines, which still actually have (electrical) transmission line properties but usually operate at 50 Hz or 60 Hz. The wavelength in air at 50 Hz is 6000 km so the lines must be very long before the distributed effects become significant.
[2] The transmission will also have *mutual* inductance but the self inductance usually dominates.
[3] The ‘skin effect’ is the tendancy for alternating current fields in conductors to penetrate smaller and smaller depths as the frequency is increased.
[4] A transmission line (singular) includes conductors for both the ‘forward’ and ‘return’ current paths, in this case just two conductors. ‘Forward’ and ‘return’ paths are often also used for AC to refer to physical lines lines relative instantaneous currents. More conductors will be considered later.



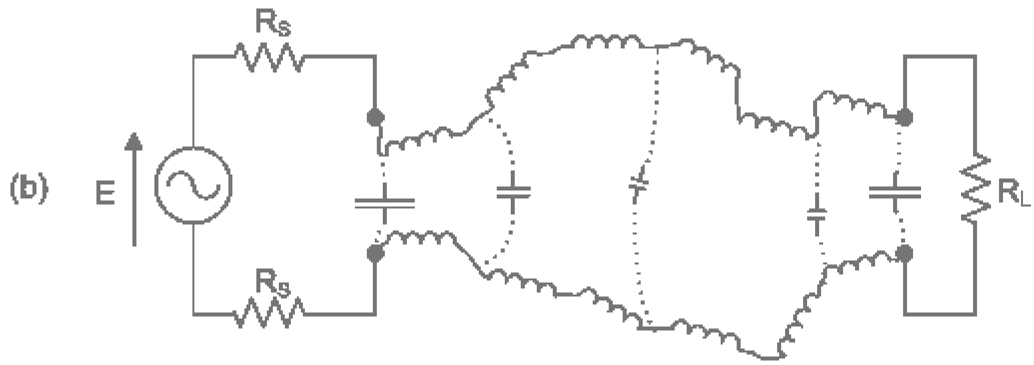


Figure 1-1 A simple DC (a) and AC (b) circuit to transfer power from the source on the left to the load on the right. The reactive components become more influential in (b) as the frequency is increased.

For the AC case[5], we know that the formulas for inductive reactance magnitude (X_L) and capacitive reactance magnitude (X_C), assuming perfect components, are:

$$X_L = 2\pi fL \quad \Omega \tag{2.1}$$

and

$$X_C = \frac{1}{2\pi fC} \quad \Omega \tag{2.2}$$

where f is the frequency in hertz (Hz), L is the inductance in henries (H) and C is the capacitance in farads (F).

Referring to each of the reactive elements shown in Figure 1-1(b), if the frequency of the AC source is increased, from (2.1) we can see that X_L will *increase in direct proportion*.

[5] Radio frequency (RF) is a frequency range subset of alternating current (AC). The frequency range of RF is open for discussion but is generally regarded to range roughly from the extent of audio frequencies (20 kHz) to where the capabilities of the latest technology dictates, currently around 300 GHz.

Similarly, from (2.2) the capacitance elements *between* the lines allows X_C to reduce in direct proportion[6]. The net effect of these changes may result in much of the power from the source being shunted before it reaches the load.

Transmission line (TL) theory has been studied extensively. One of its conclusions is that a good TL requires that the line is designed to have consistent and appropriate elements of series inductance per unit length and parallel capacitance per unit length. This means that the forward and return conductors must be parallel and their spacing must be relatively small compared to the length of the line. Another is that TLs work in the most predictably when the operating frequency allows the wavelength to be substantially greater than the transverse dimensions of the TL. Exceeding this requirement may allow the creation of higher order, non transverse electric magnetic (non-TEM) modes or waveguide modes which the TL was not designed for[7]. References on transmission line theory are widespread [1] [2] [3], but we will not look at the theory from first principles but more their practical implementations for high frequency and high speed operation[8].

1.3 The Simple Two Conductor Balanced Transmission Line

The higher the frequency of operation, the more important it is to design the TL correctly for important electrical parameters such as characteristic impedance, loss per unit length and spatial phase shift[9]. There are many types of TLs, but one of the simplest is the two conductor balanced type which is shown schematically in Figure 1-2. This comprises two straight, parallel conductors in the same plane of length L and spacing W. For a TL to be balanced, there must be a plane of symmetry bisecting the conductors and perpendicular to the plane that contains the lines. This TL is designed with parallel forward and return lines[10] of critically chosen dimensions and electrical properties, both of the conductors themselves and the dielectric material which both supports them physically and influences the field between them. If the spacing between the lines is W and the length L as shown, then the TL would only be useful if $W \ll L$. In this case the source comprises a balanced Thevenin equivalent circuit with a source impedance of $R_S/2$ in each leg of the balanced line and a load impedance of R_L [11].

[6] Or follow a reciprocal relationship to frequency. For real, as opposed to ideal components, several secondary effects become significant at higher frequencies.

[7] Here we are considering TLs which support TEM waves, not waveguide transverse electric (TE) modes or transverse magnetic (TM) modes.

[8] High *speed* refers to a digital signalling circuit and is measured in bits per second (bit/s). High frequency refers to a sinusoidal (analog) voltage-time waveform and is measured in cycles per second or hertz (Hz).

[9] Spatial phase shift is the phase shift on account of position along the TL at a fixed time, as distinct from temporal phase shift which is the phase shift as related to time at a fixed position on the TL. *Instantaneous* phase shift accounts for both spatial and temporal phase shift. Often, simplifications can be made which allow us to mathematically ignore either type.

[10] As RF is an alternating current the terms 'forward' and 'return' are not literally the long term steady state currents as with DC but mean that, at any instant, one wire will carry the forward current and the other the reverse current, and vice versa.

[11] We often refer use source and load *impedances* interchangeably with source and load *resistances*. Here we are only considering the reactive properties of the TL itself so, in the case of the sources and load, they are equivalent.

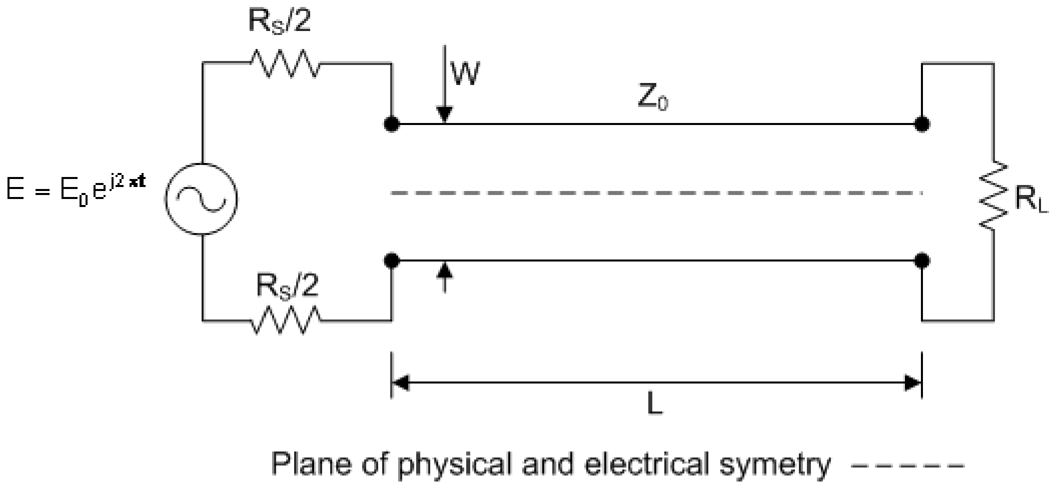


Figure 1-2 A schematic of a simple balanced transmission line comprising two straight parallel wires of spacing W and length L: one for the 'forward' current and the other for the 'return' current

The TL shown in Figure 1-2 may be represented by its equivalent circuit elements (inductance, capacitance, resistance and conductance) as shown in Figure 1-3. These are represented by the symbols L, C, R and G respectively.

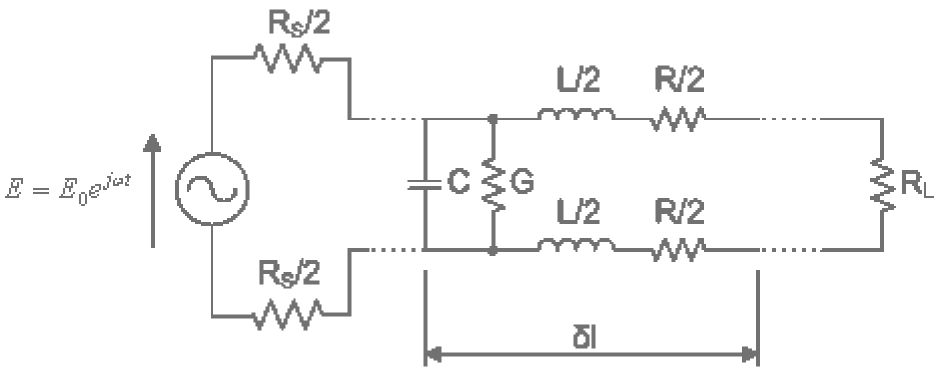


Figure 1-3 The schematic for a two conductor lossy balanced transmission line comprising series elements of inductance and resistance with shunt elements of capacitance and conductance

A very small elementary length δl is chosen, so $\delta l \ll L$. Each section has the following electrical properties:

- a series inductance of L henries per metre (H/m);
- a parallel capacitance of C farads per metre (F/m);
- a series resistance of R ohms per metre (Ω/m);
- a parallel conductance of G siemens per metre (S/m).

The symbol representations for L, C, R and G differ from the more common usage in that *they are not absolute values but refer to 'per unit length' quantities*. In this example we have used the SI length unit of one metre (m). Therefore, as the TL may be lengthened by cascading further identical sections, the per unit length values are linearly additive.

The inclusion of finite values of R and G implies that it is a *lossy* transmission line. A lossy TL is one which attenuates the signal passing through it. The greater the attenuation, the more lossy the line. Loss is usually undesirable but is present in all practical TLs and most systems require it to be minimised. However, the assumption that a line is loss-free is often sufficiently accurate to a first approximation as it allows several simplifications.

In Figure 1-3 we have represented the behaviour of the voltage source using the Thevenin equivalent by denoting the (sinusoidal) voltage as follows:

$$E = E_0 e^{j\omega t} \tag{2.3}$$

This expression uses the complex exponential format to describe a sinusoidal voltage, where:

- E is the instantaneous voltage in volts (V) ;
- E_0 is the sinusoidal peak voltage (or amplitude) in volts (V);
- ω is the angular frequency in radians per second (rad / s);
- t is the time in seconds (s);
- j is the complex coefficient ($j = \sqrt{-1}$).

The frequency (f) in hertz (Hz) is related to ω by:

$$\omega = 2\pi f \tag{2.4}$$

The complex exponential format may be derived from the rectangular trigonometric format by using one of Euler's relationships [7] [8]:

$$e^{jx} = \cos x + j \sin x \tag{2.5}$$

Therefore, taking the example in Figure 1-3,

$$\begin{aligned} e^{j\omega t} &= \cos \omega t + j \sin \omega t \\ E &= E_0 e^{j\omega t} \\ &= E_0 \ \ \cos \omega t + j \sin \omega t \end{aligned} \tag{2.6}$$

A complex exponential representation of a sinusoidal source such as $E = E_0 e^{j\omega t}$ represents the *real part* of the complex expression on the right hand side so the full expression should strictly be $E = \mathrm{Re}(E_0 e^{j\omega t})$, where the operator $\mathrm{Re}(z)$ means the real part of the complex argument z .

1.4 Common Transmission Line Parameters

The following sections describe some of the common TL parameters derived from TL theory [8] [9].

1.4.1 Characteristic Impedance

The characteristic impedance (Z_0) of a TL has units of ohms (Ω) like other impedances. It is a measure of perhaps its most fundamental electrical property of a TL given by the following equation:

$$Z_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}} \tag{2.7}$$

Most TLs are designed for operation at relatively high frequencies and for the lowest possible loss[12]. The series resistance (R) and parallel conductance components (G) in (2.7) are therefore minimised and the electrical characteristics are chosen such that:

$$\omega L \gg R \tag{2.8}$$

and

$$\omega C \gg G \tag{2.9}$$

Therefore, for the loss free case:

$$Z_0 = \sqrt{\frac{j\omega L}{j\omega C}} = \sqrt{\frac{L}{C}} \tag{2.10}$$

(2.10) is often an adequate approximation of the characteristic impedance of a TL designed to operate at RF[13]. The characteristic impedance of a TL must be matched to both the source impedance and the load impedance for optimum transfer of power, without reflections, from the source to the load

1.4.2 The Matched Loss Free TL: Spatial Phase Constant

From (2.3) the expression in complex exponential notation, for a sinusoidal voltage against time waveform, such as from an oscillator, is:

$$E = E_0 e^{j\omega t} \tag{2.11}$$

This is a time varying expression equivalent to a co-sinusoidal voltage against time waveform using the symbols that were defined in Section 1.3. Here, the coefficient of the

[12] If the TL is operated at relatively low frequencies, for example audio, the inequalities in (2.8) and (2.9) can become invalid and Z_0 may vary significantly with frequency.

[13] The RF range is not formally defined but with today's technologies is widely understood to cover approximately from the highest audio frequency (about 20 kHz) to around 300 GHz.

complex exponential (ωt) is the *temporal* (time related) phase. For the loss-free TLs we also need to know how the voltage against time waveform changes *with position along the line in the direction of propagation*. We will define this by the variable z so, for example, $z = 0$ at the beginning of the line and $z = l$ at a distance l from the beginning of the line. We will consider the beginning of the line to be on the left and the wave to propagate within the line from left to right. To represent the *spatial* phase (say ϕ radians) we need to include this in the complex exponential coefficient, so this becomes $\omega t - \phi$. Therefore, the expression for the instantaneous voltage, which includes components for both the temporal and spatial phase, becomes:

$$E = E_0 e^{j(\omega t - \phi)} \tag{2.12}$$

For a uniform loss free TL, the *spatial* phase shift ϕ is a linear function of both the wavelength within the TL itself and of the position along the line. For the convention described, the negative sign is a result of the retardation of phase as the wave moves to the right. We will consider a fixed frequency, so that the wavelength is also fixed. The number of wavelengths per unit length is represented by a quantity β known as the spatial phase constant, normally expressed in radians per metre (rad / m). Therefore

$$\beta = \frac{2\pi}{\lambda} \tag{2.13}$$

where λ is the wavelength *within the TL*, and the *spatial* phase is given by

$$\phi = \beta z \tag{2.14}$$

Substituting for ϕ from (2.14) into (2.12) gives:

$$E = E_0 e^{j(\omega t - \beta z)} \tag{2.15}$$

We know that ωt is the temporal phase. In many TL problems the line is excited by steady state fixed frequency waveforms and with these the temporal phase exponential may be replaced with unity[14]. Therefore, considering only the spatial phase:

$$E = E_0 e^{-j\beta z} \tag{2.16}$$

Provided the frequency is fixed, the $e^{j\omega t}$ term is *understood to be unity*. So far we have only considered loss free TLs because this allows us to use several mathematical simplifications. In fact many practical TLs have sufficiently small loss to often make this a reliable approximation.

1.4.3 The Matched Loss Free TL: Loss Per Unit Length

[14] To summarise, we are considering their operation in the frequency domain. For the time domain we would need to use conversions, such as Laplace or Fourier and that would be a whole new subject.

The solution of the lossy TL differential equations for current and voltage with respect to time and position require a complex term known as the *propagation* constant γ, where

$$\gamma = \alpha + j\beta \tag{2.17}$$

α is the *attenuation* constant in Nepers per metre (Np/m) and we noted in the last section that β is the phase constant in radians per metre (rad/m). So the instantaneous voltage at some point, a distance z along the line, is given by:

$$\begin{aligned} E &= E_0 e^{-\gamma z} \\ &= E_0 e^{-(\alpha + j\beta)z} \\ &= E_0 e^{-\alpha z} e^{-j\beta z} \end{aligned} \tag{2.18}$$

The Neper is a logarithmic measure of power ratio, similar to the decibel (dB), but defined in terms of the *natural* logarithm of the linear power ratio, unlike the decibel which uses 10 times the logarithm to base 10. Therefore:

$$Np = \ln\left(\frac{P_{out}}{P_{in}}\right) \tag{2.19}$$

where P_{OUT}/P_{IN} is the linear ratio of the output lower divided by the input power, each using the same linear unit, such as watts.

In terms of the per unit parameters for a realistic (lossy) TL that was considered in Figure 1-3, the TL theory provides the following [8] [9]:

$$\gamma = \sqrt{R + j\omega L \quad G + j\omega C} \tag{2.20}$$

Thus, if R, L, G and C are known, γ may be calculated and the individual values for α and β determined by equating the real and imaginary coefficients of the result with those of (2.17).

Sometimes, even with finite but small loss, it is sufficiently representative to consider the loss free case, where $R = 0$ and $G = 0$. In this case

$$\gamma = \alpha + j\beta = \sqrt{0 + j\omega L \quad 0 + j\omega C} = j\omega\sqrt{LC} \tag{2.21}$$

Therefore, using (2.17), $\alpha = 0$ as there is no real component and

$$\beta = \omega\sqrt{LC} \tag{2.22}$$

1.4.4 Characteristic Impedance Representations for Various Geometries

Figure 1-4 schematically shows the characteristic impedances associated with four simple TL geometries, three of which include the influence of a nearby ground plane[15]. (We will see later that it is often very difficult for a TL to *avoid* the influence of a ground plane). Each case is assumed to be loss-free so there are no per unit length components of resistance (R) or conductance (G), only capacitance (C) and inductance (L). We know from (2.7) and (2.10) that characteristic impedance has units of ohms just like a resistor so for each geometry, every component of characteristic impedance is represented by a resistor symbol. However, these are not resistors in the sense of lumped components but characteristic impedances.

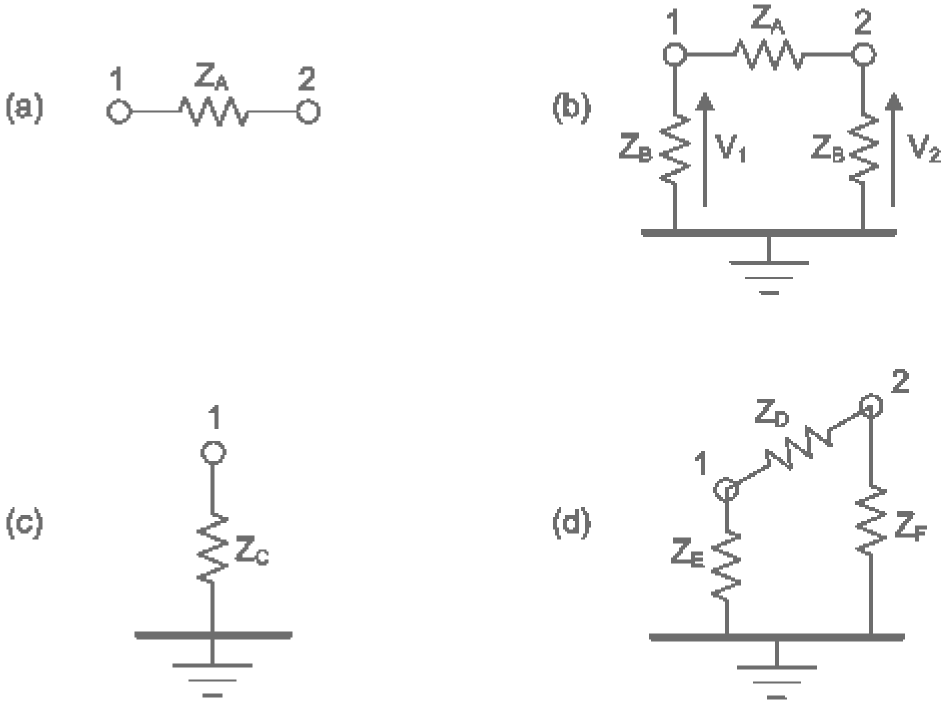


Figure 1-4 Schematic representations of characteristic impedances for various transmission line geometries: (a) 2 wire balanced, (b) 2 wire balanced with ground, (c) single ended (unbalanced) and (d) 2 wire unbalanced (asymmetric). In each case we are considering a plane perpendicular to the direction of propagation

Figure 1-4(a) represents a parallel 2 wire TL, like those shown in Figure 1-2 and Figure 1-3, with the forward and return wires identified as 1 and 2[16]. This is also a form of balanced line like that shown in Figure 1-3. The wires individually will possess self inductance per unit length (L) and are sufficiently close to have coupling between them resulting in a capacitance per unit length (C). In this case there is no further coupling and, using (2.10), the characteristic impedance between them Z_A is given by:

$$Z_A = \sqrt{\frac{L}{C}} \qquad (2.23)$$

[15] The ground plane has to be sufficiently close to allow coupling between it and the TL. The higher the operating frequency the greater this will be.

[16] In fact due to the balanced symmetry it makes no difference which wire is the forward and which the return.

In practice it is rare that we find a balanced line that is not influenced in some way by one or more other nearby conductors or conductive surfaces such as ground planes. Figure 1-4(b) is an example which comprises a two conductor balanced line (1 and 2), otherwise as considered in Figure 1-4(a), but horizontally orientated above a ground plane and sufficiently close to create coupling. In this case we have the component Z_A between the wires 1 and 2, and two new further identical components Z_B between each wire and ground. Often, after attempting to design a balanced TL like that shown in Figure 1-4(a), this is the type which results. Although it is represented here as two wires above a ground plane, the same schematic applies to other cases where the ground connection influences the conductors such as the following:

- Coupled microstrip (PCB), Figure 2-2(b).
- Coupled stripline (PCB), Figure 2-2(e).
- Shielded twisted pair, Figure 2-2(h).
- Shielded wires with cylindrical ground, Figure 2-3 and Figure 2-4.

Figure 2-5 is a three dimensional schematic for Figure 1-4(b) which shows the elements of self inductance and capacitance.

Examples such as these have become widespread over recent years to carry high speed digital signalling, *designed to be excited in perfectly differential mode*. We will see in chapter 4 that in these cases the ground plane although present close to the conductors, ideally should not carry any signalling current. This type of excitation is very desirable for electromagnetic compatibility (EMC) reasons.

Figure 1-4(c) applies to the very common single ended (SE) case in which the forward and return lines comprise a single wire and the ground connection, so *ground currents always flow*. Some examples which may be represented in this ways are:

- 'Stand-alone' microstrip, Figure 2-2(a).
- Uncoupled microstrips (Figure 1-4(c) for each microstrip), Figure 2-2(c).
- Coaxial cable, Figure 2-2(f).

Figure 1-4(d) represents a two wire TL above a ground plane but asymmetrically disposed and therefore unbalanced. Rarely are TLs intentionally constructed like these but this type has been analysed by [TBA].

1.4.5 Characteristic Impedance Definitions for 2 Wire Balanced Lines Above a Ground Plane

Section 1.4.4 described how the TL configuration shown by the characteristic impedance elements in Figure 1-4(b) has a number of advantages in today's high speed digital designs, particularly when it is excited in *purely differential mode*. This is achieved when one line is excited with the same magnitude but *exactly in anti-phase* (180º or $\boldsymbol{\pi}$ radian) to the other line. Referring to Figure 1-4(b) therefore, if the phasor voltages $\boldsymbol{V}_1$ and $\boldsymbol{V}_2$ on lines 1 and 2 respectively are measured relative to a common reference, in this case the ground connection, then for perfectly differential excitation:

$$|\boldsymbol{V}_1| = |\boldsymbol{V}_2| \tag{2.24}$$

and

$$\angle \boldsymbol{V}_1 - \angle \boldsymbol{V}_2 = \boldsymbol{\pi} \tag{2.25}$$

Unfortunately, this is often difficult to achieve and maintain along the length of the line and there is often some unwanted level of *common mode* component. Unfortunately, the impedance of a TL like that shown in Figure 1-4(b) differs according to how it is excited. Therefore, there are several characteristic impedance definitions for symmetric balanced TLs as follows:

- Single Ended Impedance;
- Odd Mode Impedance;
- Even Mode Impedance;
- Differential Mode Impedance;
- Common Mode Impedance.

These are described in the following sections.

1.4.5.1 Single Ended Impedance

The single ended impedances of wire 1 relative to ground (Z_{SE1}) and wire 2 relative to ground (Z_{SE2}) are equivalent as the line is balanced, given by the following expressions:

$$Z_{SE1} = Z_{SE2} = Z_B \,||\; Z_A + Z_B \; = \frac{Z_B \;\; Z_A + Z_B}{Z_A + 2Z_B} \tag{2.26}$$

If the separation between lines 1 and 2 was increased sufficiently to make the coupling between them negligible, with each still maintaining the same coupling to ground, then Z_A approaches infinity, (2.26) may be re-written:

$$Z_{SE1} = Z_{SE2} = \frac{Z_B \;\; Z_A + Z_B}{Z_A + 2Z_B} = \frac{Z_B\left(1 + \frac{Z_B}{Z_A}\right)}{1 + \frac{2Z_B}{Z_A}} \tag{2.27}$$

Thus when $Z_A = \infty$, $Z_{SE1} = Z_{SE2} = Z_B$. This is equivalent to two uncoupled single ended lines, each of which has a characteristic impedance of Z_B. In fact, each of these may be represented as shown in Figure 1-4(c), in this case with characteristic impedance Z_C.

In Figure 1-4(c) simply $Z_{SE} = Z_C$.

1.4.5.2 Odd Mode Impedance

The odd mode impedance, Z_{OM} is given by:

$$Z_{OM} = Z_{OM1} = Z_{OM2} = \frac{Z_A}{2} \,||\, Z_B = \frac{Z_A Z_B}{Z_A + 2Z_B} \tag{2.28}$$

1.4.5.3 Even Mode Impedance

The even mode impedance, Z_{EM} is given by:

$$Z_{EM} = Z_{EM1} = Z_{EM2} = Z_B \tag{2.29}$$

1.4.5.4 Differential Mode Impedance

The differential mode impedance, Z_{DM} is given by:

$$Z_{DM} = Z_A \,||\, 2Z_B = \frac{2Z_A Z_B}{Z_A + 2Z_B} \tag{2.30}$$

Therefore, from (2.28), Z_{DM} is twice the odd mode impedance Z_{OM} so

$$Z_{DM} = 2Z_{OM} \tag{2.31}$$

1.4.5.5 Common Mode Impedance

The common mode impedance, Z_{CM} is given by:

$$Z_{CM} = Z_B \,||\, Z_B = \frac{Z_B}{2} \tag{2.32}$$

Therefore, from (2.29), Z_{CM} is one half the even mode impedance Z_{EM} so

$$Z_{CM} = \frac{Z_{EM}}{2} \tag{2.33}$$

2. BALANCED AND UNBALANCED LINES

2.1 Connections to Ground and Screening

Usually in RF circuits it is very common and convenient to have a ground plane: a large area of a good conductor surface which provides a common low impedance path and acts as a reference point.[17] This improves electromagnetic compatibility issues and also can serve as

[17] The expression 'ground' has persisted from the early days of electricity when a connection literally to the ground provided a return path for an electrical circuit, even one carrying considerable power, thus effectively

the return path for RF currents[18]. In fact, it is very difficult to design RF circuits which do not either need a ground plane or are heavily influenced by nearby ground planes, intentionally or otherwise.

One example of such a ground 'plane' in the form of a cylinder is the outer conductor of a coaxial transmission line, shown schematically in Figure 2-1 which is connected to a Thevenin source at one end and load at the other.

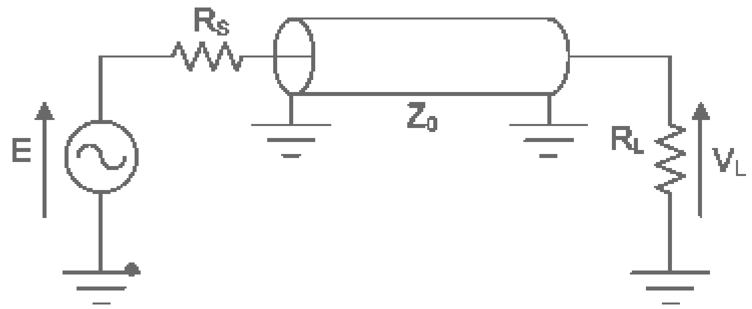


Figure 2-1 Circuit schematic of a Thevenin equivalent voltage source with source impedance R_S feeding a load of impedance R_L via coaxial transmission line with a characteristic impedance Z_0

Figure 2-2(f) shows a cross section of the coaxial transmission line. There are two reasons for the screen in a coaxial transmission line[19].

- to prevent radiated emissions from the transmission line which might interfere with other equipment;
- to suppress radiated interference which might be received from other equipment and couple to the transmission line.

halving the length of wire required. Today almost every RF device, even mobile handsets, have one or more 'ground-planes' but of course are not literally connected to the ground.

[18] Planes or enclosures, usually connected to 'ground' are usually designed to minimise radiated emissions from the circuit and/or to minimise the circuit's susceptibility to externally generated radiation.

[19] In RF engineering it is rarely possible to screen anything *perfectly.* However, with a well chosen conductor, thickness, electrical properties (conductivity and relative permeability) and physical type (preferably tubular as opposed to braid), very high levels of screening often can be achieved.

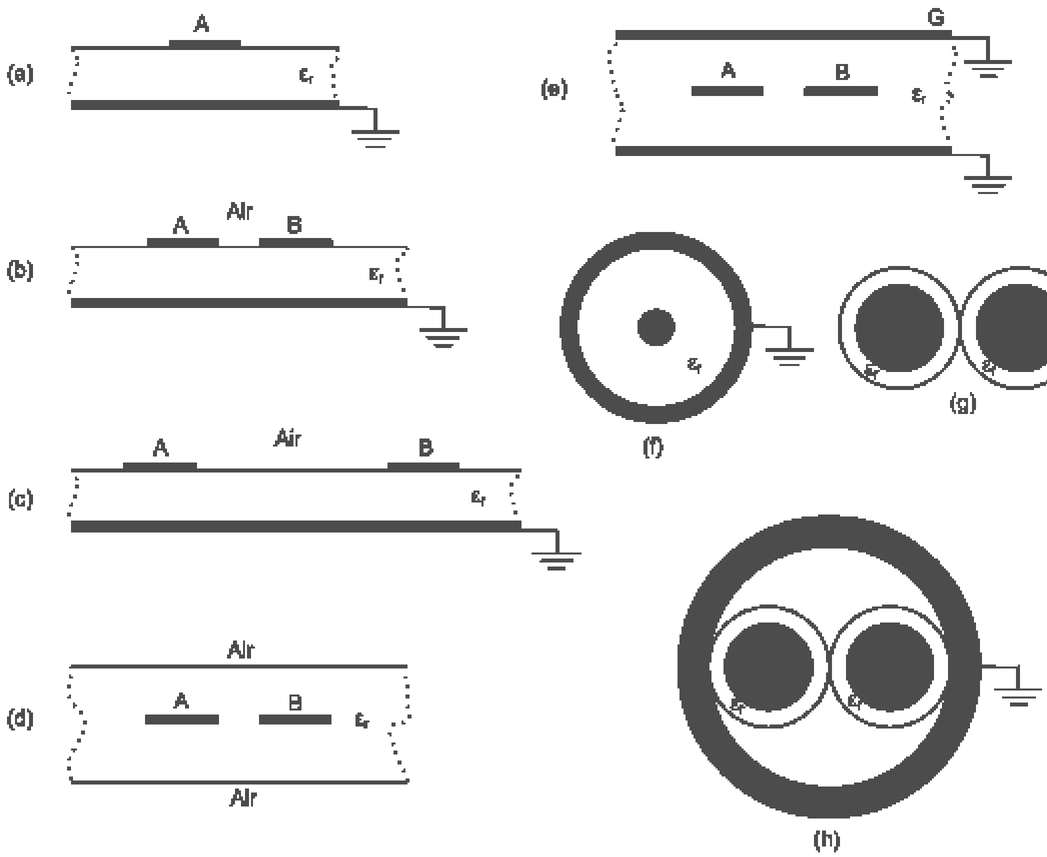


Figure 2-2 Cross sections of various single ended and balanced transmission lines in the forms of PCBs and cables

One example of how a transmission line screen might be used is shown in Figure 2-3. In this case, the transmission line itself is balanced and the whole line is completely surrounded by a cylindrical conductor connected to ground[20].

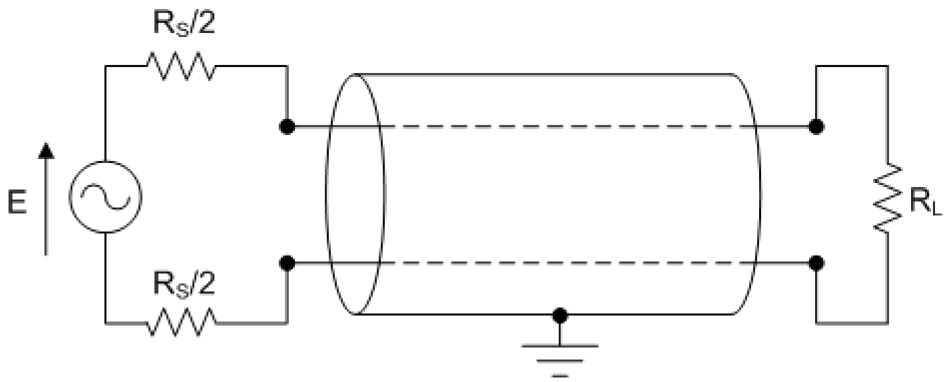


Figure 2-3 An illustration of how a cylindrical screen, usually connected to ground, might be necessary to surround a balanced transmission line

The influence of the ground connection considered alone depends on several EMC aspects such as the frequency of the interference. Normally the screen would be of relatively small diameter, perhaps just slightly larger than the spacing of the signal wires themselves, such

[20] This is not of course a coaxial line because none of the conductors shares a common axis.

as the example shown in Figure 2-2(h). A common form of this type of TL is used for wired ‘Ethernet’ network connections and this construction is commonly known as ‘shielded twisted pair’ (STP)[21]. If it is sufficiently close to the TLs it will create elements of capacitance coupling between the screen itself and each of the main conductors. This will influence the characteristic impedance between the lines. To help describe this, Figure 2-4 shows a schematic cross section of the line which includes the capacitance elements. As the metallic part of the screen itself has a relatively large cross section compared to the conductors, it has low impedance and is commonly approximated this to zero impedance.

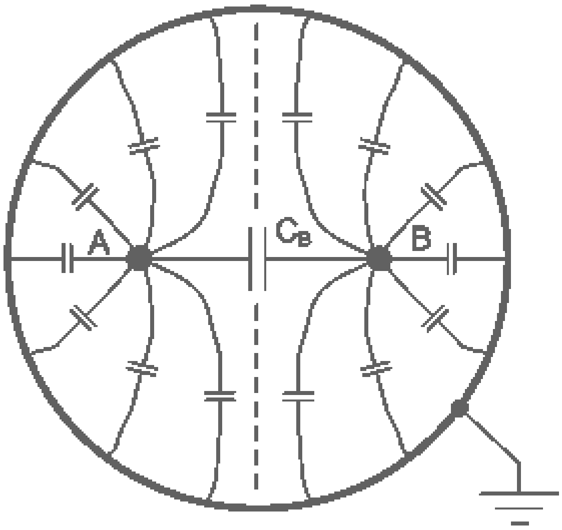


Figure 2-4 A schematic cross section of a balanced two wire transmission line comprising wires A and B, together with a coupled cylindrical screen enclosing both and connected to ground.

So now we have, not only the forward and return wires 1 and 2, but a third conductor which is also connected to ground. The presence of the capacitive coupling between either conductor and ground means that we have effectively a total of three components of characteristic impedance:

- between line 1 and line 2;
- between line 1 and ground;
- between line 2 and ground.

These may again be represented schematically as shown in Figure 1-4(b). As well as a characteristic impedance Z_A between line 1 and line 2, there will be additional and equal characteristic impedances (Z_B) between line 1 and ground and another between line 2 and ground.

A schematic way of representing this line in three dimensions is shown in Figure 2-5. The line is balanced (there is an imaginary plane of symmetry midway between the lines and perpendicular to the plane containing the lines). Also, the ground connection passes through the plane of symmetry. The elements of capacitance (C_S) from each line to ground are the

[21] The only reason for twisting the TL pairs in an Ethernet STP cable is to simplify manufacture thereby reducing costs.

equivalent values for the similar elements shown in Figure 2-4. The elements of capacitance between the lines AA' and BB' are denoted C_B as shown in Figure 2-4. Figure 2-5 also shows the elements of series inductance (L_P) which represent the inherent self-inductance of the wires themselves. As with the screened version, we have assumed that the impedance of the ground connection is zero.

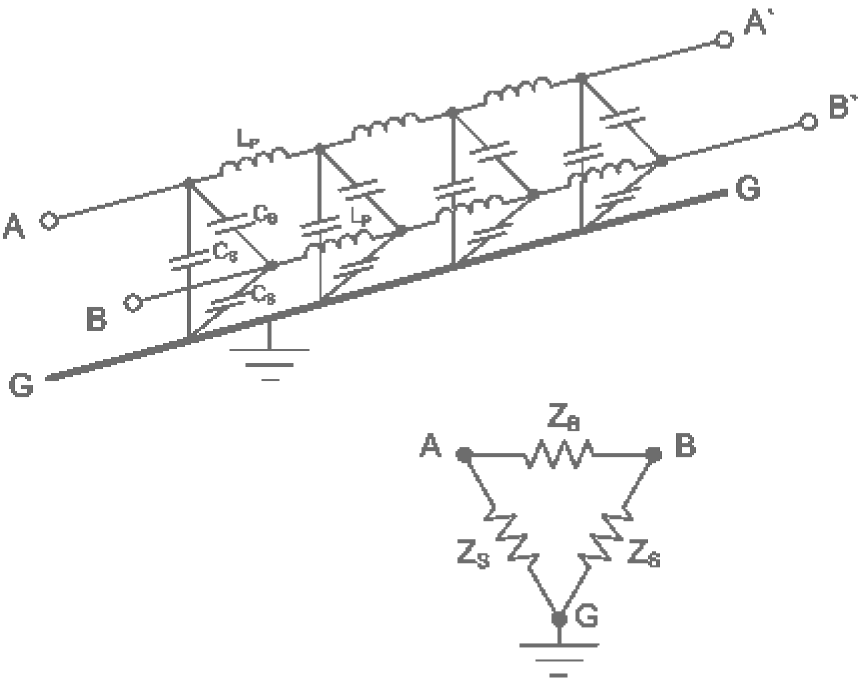


Figure 2-5 Schematic of the electrical elements of a balanced transmission line

2.2 Excitation Modes

2.2.1 Differential Mode Excitation of Balanced Lines

Differential excitation of balanced lines is an important choice in terms of electromagnetic compatibility (EMC) because any components of radiated noise coupled into the line from external sources normally occur in the common mode. The reciprocal argument also applies: a differentially excited TL will radiate less than one with similar geometry but excited in common mode[22]. The EMC arguments are intuitively clear from Figure 2-7 which represents a typical differentially excited TL because $W \ll L$. So the individual conductors are relatively closely spaced compared to typical TL lengths. Indeed, in recent years, as digital signalling speeds have increased, there has been a general move in this direction away from single ended lines TLs[23]. Figure 1-2 shows another schematic of a balanced lines excited differentially. *Perfect* differential excitation applies when the voltage phases at the same point on the lines forming a TL in the direction of propagation:

- have the same magnitudes, and

[22] Notice the reference to 'similar' geometry. This means using similar dimensions and construction of for example a circuit on a PCB or substrate. Additional screening may improve EMC performance for single ended lines but the extra grounded surfaces can alter the TL parameters.

[23] Depending on the geometry, balanced lines sometimes occupy less space on PCBs and substrates than the equivalent single ended lines.

- are exactly in anti-phase (180º or π radian).

The voltages must also be measured relative to the same reference point, normally the ground connection.

Figure 2-6, Figure 2-7 and Figure 2-8 show three methods of differential excitation connecting one or two sources on the left to one or two loads on the right. Figure 2-6 and Figure 2-7 utilise a single balanced (coupled) TL and Figure 2-8, two unbalanced single ended coaxial cables. Figure 2-6 is the ideal case that we generally aim for, without any influences from any ground or other connections. Unfortunately, it is often difficult to create a perfectly balanced source, like this and Figure 2-7 shows something that is more realisable[24]. In this, differential excitation is achieved using two identical sources locked to an identical frequency but with one in perfect anti-phase compared with the other. The reference point used to achieve this is normally a ground connection. Assuming perfect balanced lines and perfect differential excitation, the ground connections effectively become redundant as there will be no ground currents.

Figure 2-6 and Figure 2-7 are often used for TLs used for the type of interfacing found on PCBs and the substrates used for hybrid circuits and integrated circuits.

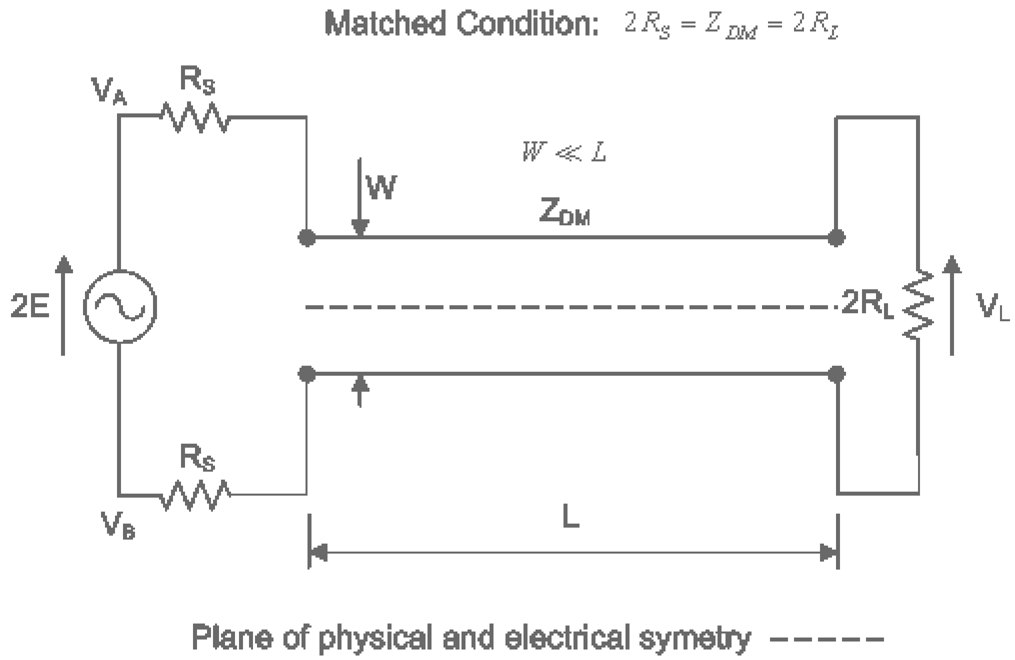


Figure 2-6 The ideal case of an open wire (coupled, balanced) transmission line with perfect differential excitation. There are no influences from any ground connections.

[24] It is difficult to create solid state sources without ground connections so they are normally single ended. One option might be to use a single ended source feeding a high specification balanced to unbalanced transformer (balun).

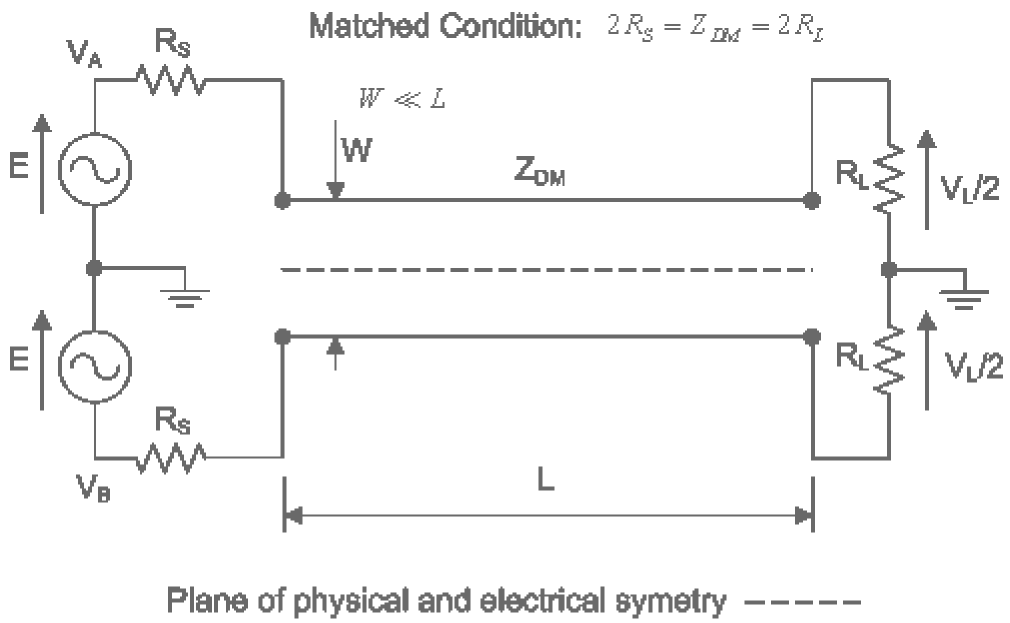


Figure 2-7 The circuit schematic for an open wire (coupled) balanced transmission line with perfect differential excitation under which no current will flow in the ground paths.

2.2.2 Differential Mode and Common Mode Excitation Using Two Separate Single Ended (Unbalanced) Transmission Lines

We often need to use cables to make signalling connections between items of infrastructure or test equipment. Particularly for high speeds, separate but electrically identical coaxial cables are used, one for each leg of the differential connection[25]. This is shown in Figure 2-8 and Figure 2-9 shows the corresponding but more unusual common mode case. This is a good example of where we require a differential connection but *we are forced to use ground connections which are formed by the screen connections of two separate coaxial cables*. For most of their applications, single ended (SE) coaxial cables are interfaced at one end with a SE source and at the other end with a SE load, as shown in Figure 2-1. Here, we have two separate but differentially excited sources, similar to those described for Figure 2-7 but the TL comprises two separate uncoupled and unbalanced TLs. The cables and associated connectors (not shown) must be *precision, delay matched types*[26].

[25] For lower speeds a lower specification twisted pair, shielded or unshielded, may be sufficient. This cable provides a single balanced transmission line with one connector at each end, for example some varieties of Ethernet cables.

[26] If they are identical cables of identical lengths then of course they will be delay matched. However, in practice there is always some finite difference in length which will manifest in a differential length and associated differential phase, for which the worst case will be at the highest operating frequency.

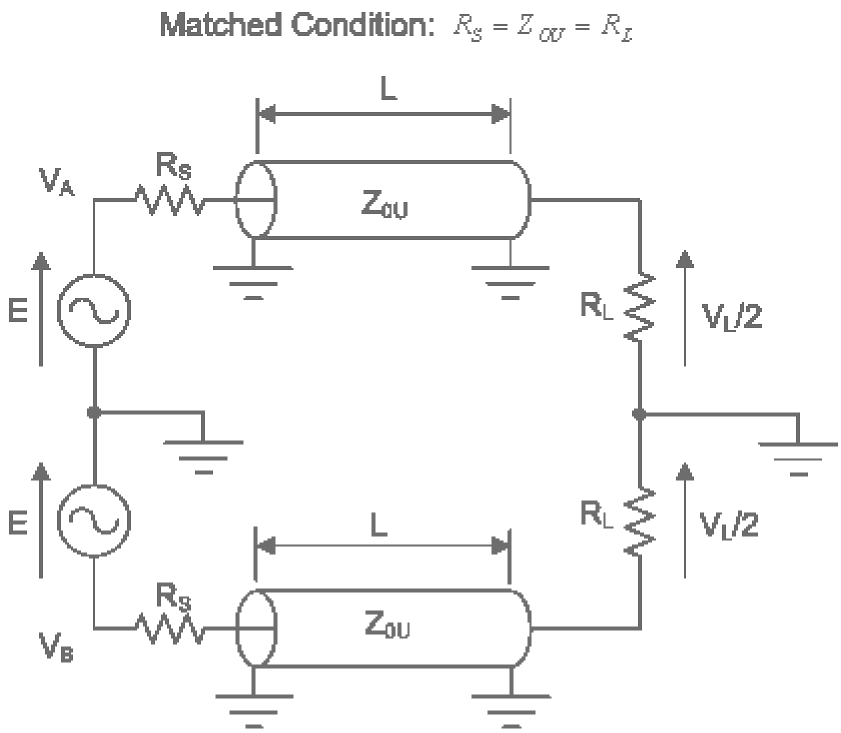


Figure 2-8 The circuit schematic for two separate coaxial lines with perfect differential mode excitation.

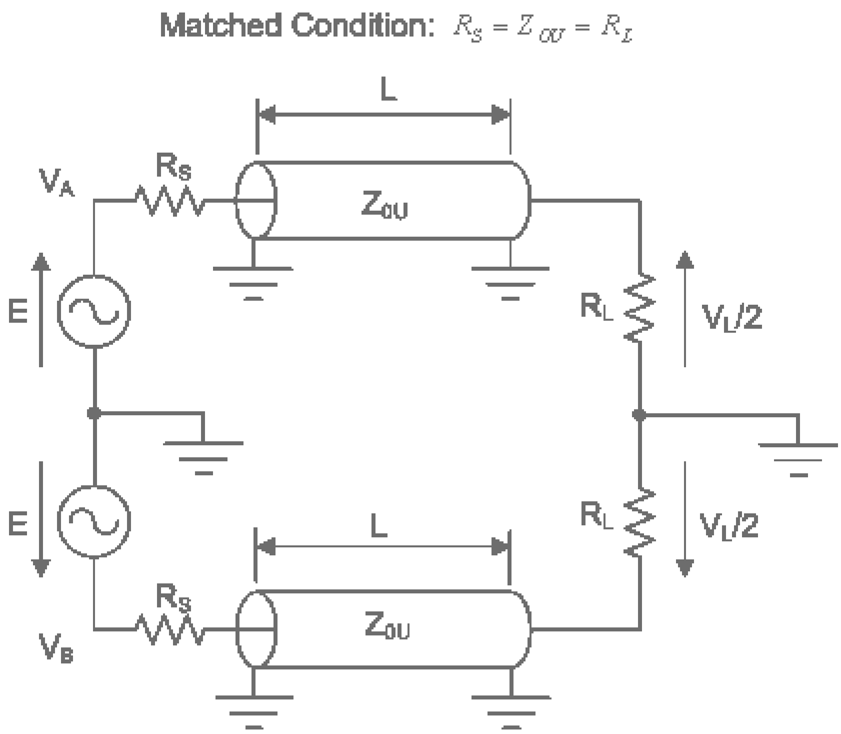


Figure 2-9 The circuit schematic for two separate coaxial lines with perfect common mode excitation.

It is interesting to consider what the differential and common mode characteristic impedances are for these configurations. This is illustrated in Figure 2-10 and in both cases the screens of the coaxial cables prevent any mutual coupling between them. This is equivalent to an infinite characteristic impedance component between them, so $R_A = \infty$ in Figure 1-4(b). Each of the other components of characteristic impedance is equivalent to the single ended characteristic impedance of each coaxial cable, so $R_B = Z_{OU}$. Therefore, we have, from (2.30) and (2.32), the following differential mode impedance (Z_{DM}) and common mode impedance (Z_{CM}):

$$Z_{DM} = 2Z_B \tag{2.34}$$

and

$$Z_{CM} = \frac{Z_B}{2} \tag{2.35}$$

For the very common case of coaxial cable with 50 Ω characteristic impedance therefore, $Z_B = 50\,\Omega$, so $Z_{DM} = 100\,\Omega$ and $Z_{CM} = 25\,\Omega$.

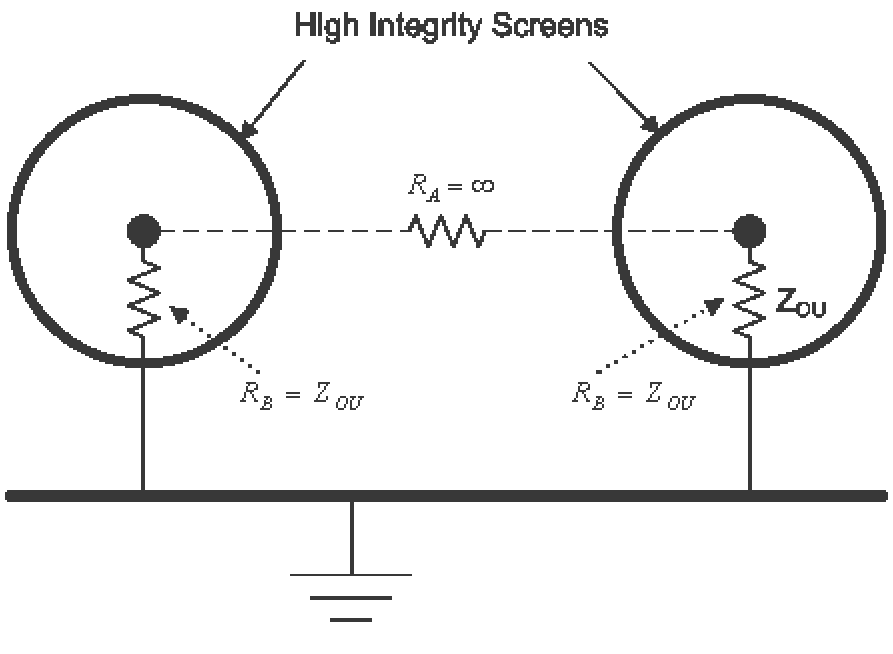


Figure 2-10 Relating the components of characteristic impedance for the connections shown in Figure 2-8 to Figure 1-4(b)

3. MATCHING AND OPTIMUM POWER TRANSFER

We noted from Section 0 that one requirement of a TL is to efficiently transfer power from one location (a source) to another (a load)[27][28]. The conditions required for optimum matching and power transfer are:

- the source impedance must be matched to the effective TL impedance[29];
- the load impedance must be matched to the effective TL impedance.

In the simple single ended example with a coaxial cable which is represented in Figure 3-1, the correctly match requirement would therefore be:

$$R_S = Z_0 = R_L \tag{2.36}$$

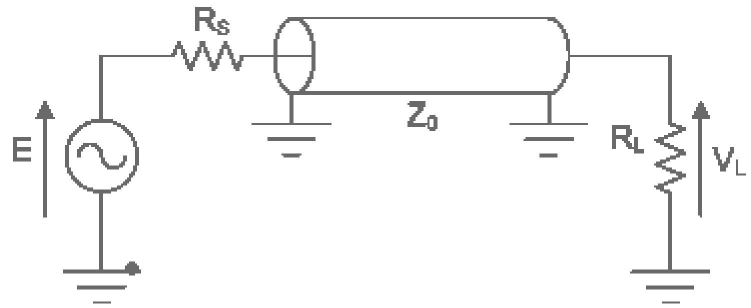


Figure 3-1 A simple single ended (coaxial) transmission line fed by a source represented as a Thevenin equivalent circuit of source resistance R_S and feeding a load resistance R_L

Figure 3-2, Figure 3-3 and Figure 3-4 show some examples of TLs designed for carrying differential signalling from a balanced source to a balanced load.

3.1 Simple Balanced Source and Load with Perfectly Differential Excitation

Figure 3-2 is perhaps the simplest case, showing a balanced source feeding a balanced load via a balanced TL which is excited differentially. This is usually what we try to achieve but unfortunately this is also the most difficult to achieve in practice due to the effects of screening, grounding and the limitations of component manufacture.

[27] Not always for maximum power transfer because there are several uses of TLs that exploit their resonant properties such as oscillators, matching circuits and filters.

[28] In many RF and high speed applications, especially at lower powers, we are less concerned with the absolute power level transferred but more important are the distortions caused by reflections at interfaces which may cause inter symbol interference and degrade the bit error rate.

[29] By effective impedance we mean that associated with the type of excitation used. For example the differential mode impedance will be associated with differential excitation.

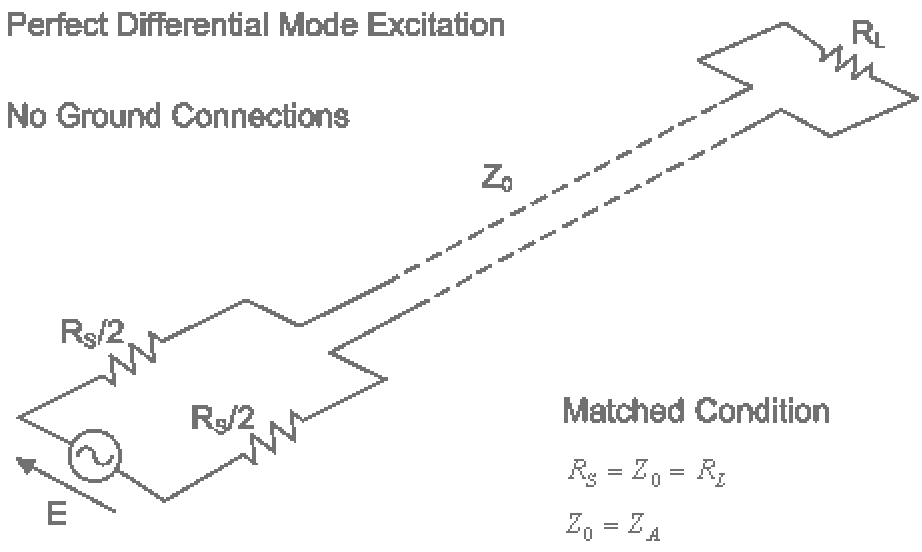


Figure 3-2 The simplest form of differentially excited connection between a balanced source, a balanced transmission line and a balanced load

The total source resistance is R_S (noting that this is a balanced source comprising $R_S/2$ in each leg). The (differential mode) characteristic impedance of the line is Z_0 and the load is R_L. The condition for correct matching is therefore

$$R_S = Z_0 = R_L \tag{2.37}$$

In more practical circuits there might screening around the TL which is likely to be grounded, and there might be other direct or indirect ground connections. Examples of these are shown in Figure 3-3 and Figure 3-4 where the TLs include overall screens for EMC purposes sufficiently close to the wires to allow coupling as we discussed in Section 2.1. The same argument would apply if the TL comprised, for example, coupled microstrips or coupled striplines both of which include groundplanes. Whilst transmission lines would function at DC, we are considering their operation at RF where even small values of coupling capacitance of a few picofarads could become significant at the higher frequencies. Notice however that, in all cases, we still have balanced symmetry throughout.

3.2 Simple Screened Balanced Transmission Line

Figure 3-3 shows a perfectly balanced Thevenin equivalent source with perfectly differential excitation feeding a balanced load via a two wire balanced transmission line *with overall screening connected to ground*. Screens of this type are usually connected to ground. If they are not there may well be unintended consequences[30].

[30] There are many EMC considerations of lines like these which can be found in other references. These are influenced by the electrical dimensions of the line. Non existent or inappropriate grounding can cause the line to radiate like an antenna and therefore be susceptible to collecting radiated interference from elsewhere.

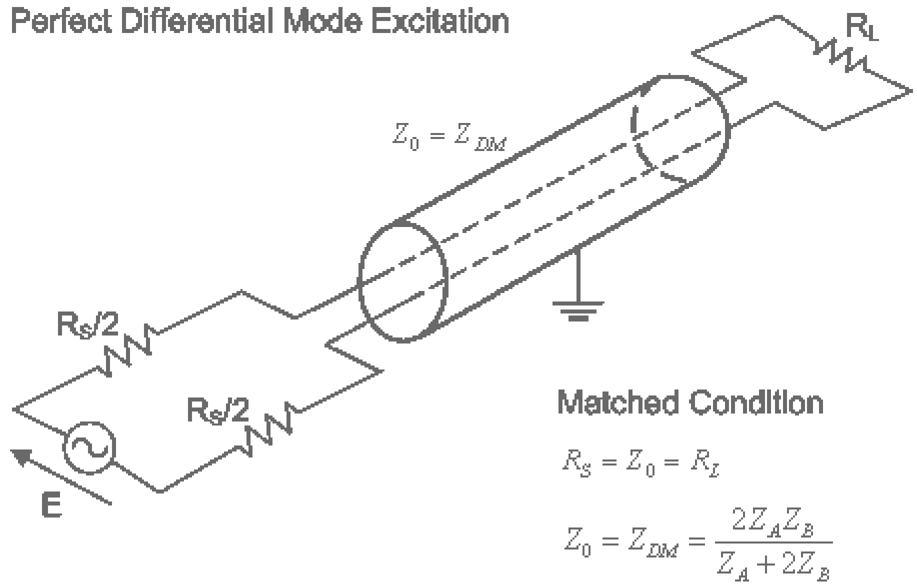


Figure 3-3 Connection of a differentially excited balanced source to a balanced load using a screened, balanced pair transmission line

If there was a direct connection between the source and load without a transmission line we know that the condition for optimum match or maximum power transfer would be:

$$R_S = R_L \tag{2.38}$$

For the TL itself, we have three separate characteristic impedances to consider as we noted in Section 1.4.5.4, Figure 1-4(b) and (2.30). As the TL is perfectly differentially excited, the relevant TL impedance that applies is the differential mode impedance Z_{DM} which we saw from (2.30) is given by:

$$Z_{DM} = Z_A \,||\, 2Z_B = \frac{2Z_A Z_B}{Z_A + 2Z_B} \tag{2.39}$$

Therefore, for perfect matching we require:

$$R_S = Z_{DM} = R_L \tag{2.40}$$

Notice from Figure 3-3 that the only ground connection shown is that of the screen itself. The source and load are both isolated from ground. Electrical engineers often refer to circuits such as these as being ‘floating’ or having no ground reference. However, strictly there is a ground reference here which has an influence, that of the screen itself.

3.3 Source and Load with Arbitrary Balanced Ground Connections

Figure 3-4 shows a more practical situation that we might have, again with a balanced source feeding a balanced load via a balanced pair with screening or other ground influence. The Thevenin equivalent voltage sources are identical, each with an identical voltage magnitude E but one of which is in anti-phase (180º or π radian) with respect to the other, indicated here by the opposing arrow directions relative to ground. For each leg, there is a series source impedance of $R_S / 2$ but also an additional shunt impedance to ground of R_{SA}.

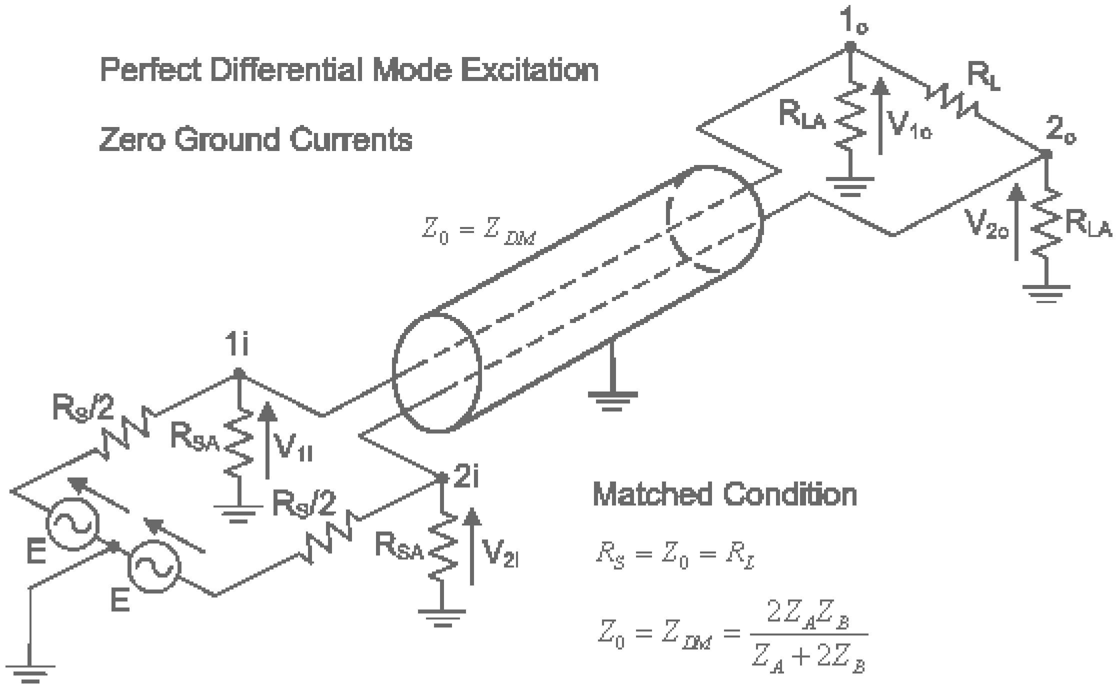


Figure 3-4 Connection of a differentially excited balanced source to a balanced load using a screened, balanced pair transmission line. Further resistors R_{SA} and R_{LA} represent typical realistic loading conditions

3.3.1 Perfectly Differential Mode Excitation

Referring again to Figure 3-4, there are two alternating current (AC) voltage sources each with one terminal connected to ground. This is another case of perfectly differential excitation because:

- the voltage source magnitudes are identical (E);
- each voltage source has a phase relationship of 180º to the other indicated by the opposing arrow directions.

Figure 3-5 shows the equivalent circuit for Figure 3-4 which separates it into three sections: source, transmission line (TL) and load. The 'Z' (impedance) symbols for the TL represent the components of characteristic impedance inherent in the TL as were defined in Figure 1-4(b).

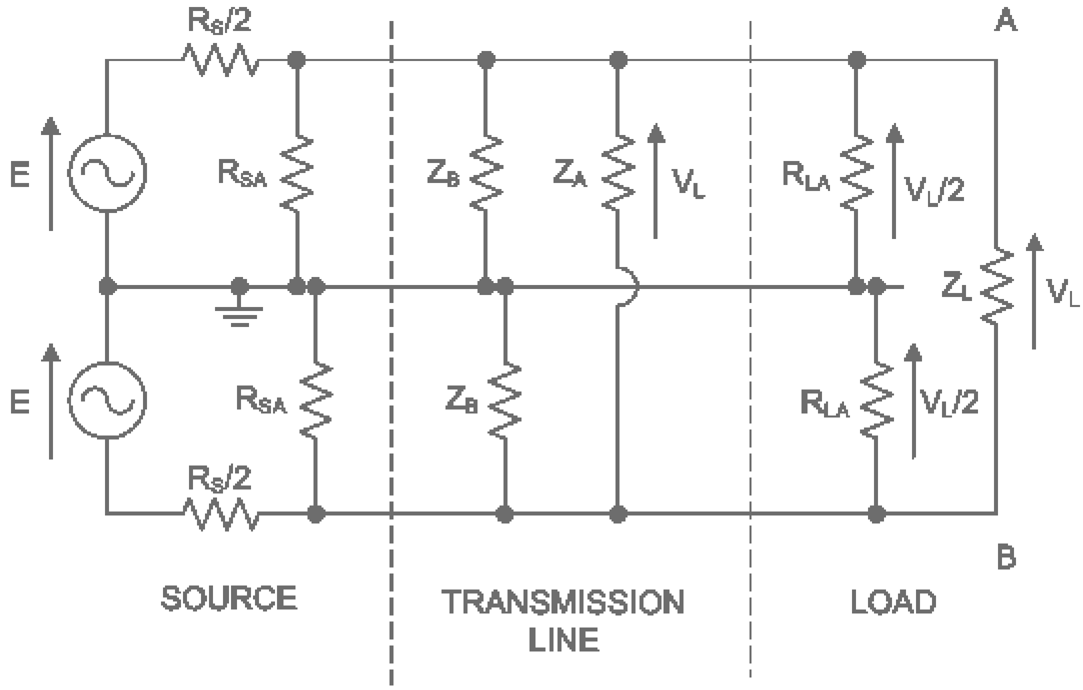


Figure 3-5 The equivalent circuit for the perfectly differentially excited balanced circuit shown in Figure 3-4

Because of the differential excitation and the balanced symmetry of the circuit, the magnitudes of the voltages at nodes A and B, are both exactly one half of the voltage across the differential load Z_L at say $+V_L/2$ but the phase relationship between them is precisely 180º, here indicated by the opposing arrow directions. In other words, the voltage at A is $+V_L/2$ and the voltage at B is $-V_L/2$. Therefore, the differential characteristic impedance component of the TL, Z_A can be divided into two equal values in series of $Z_A/2$ each, the mid-point being connected to ground. Furthermore, under this perfect condition although a ground connection is shown, there will be no current flowing through this back to the sources. Each voltage source may itself may be converted to a simpler equivalent Thevenin type. The simplified circuit is shown in Figure 3-8.

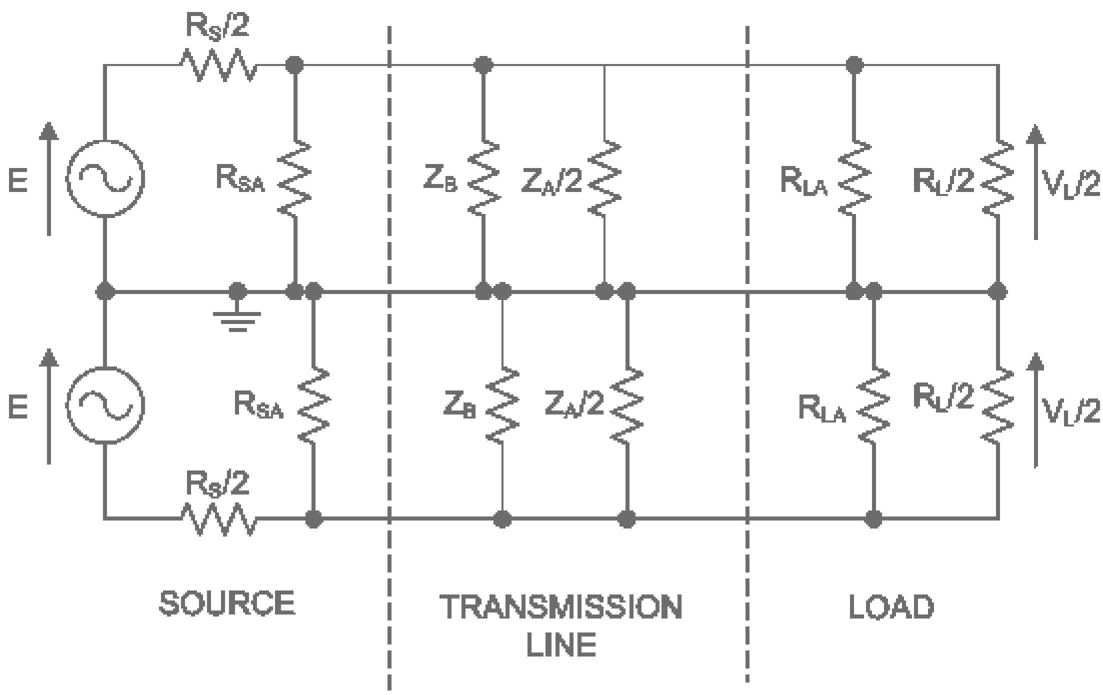


Figure 3-6 A further simplification of the circuit shown in Figure 3-5.

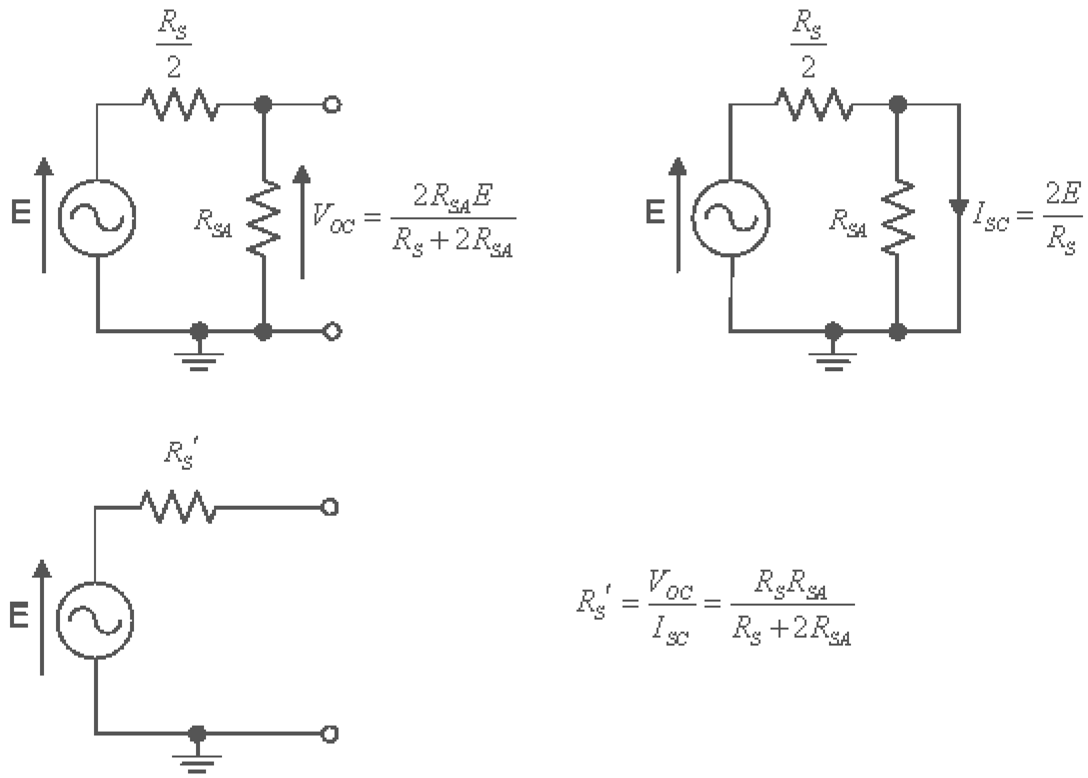


Figure 3-7 Simplification of the equivalent constant voltage source by calculating the open circuit voltage and short circuit current, hence the equivalent source resistance

R_S'

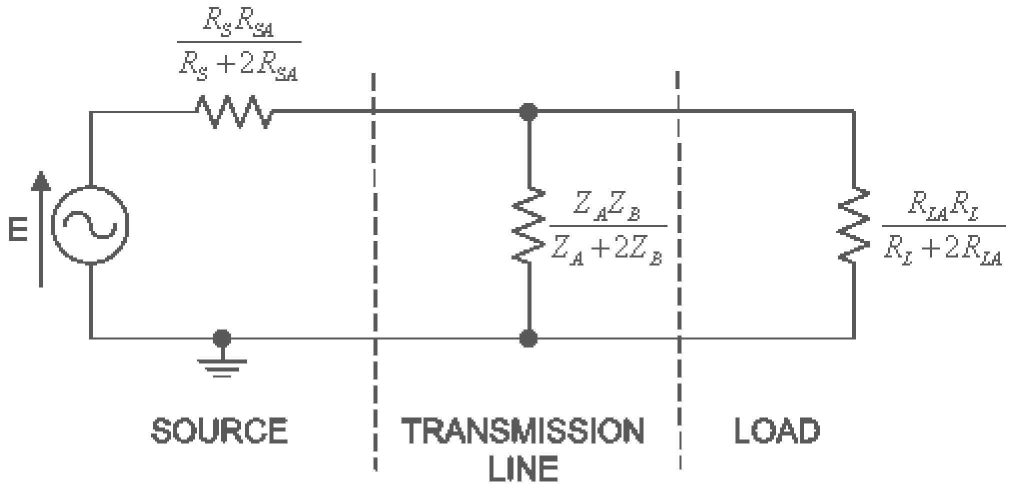


Figure 3-8 The simplest form of the circuit shown in Figure 3-5 with perfect differential excitation

From Figure 3-8, the condition for correct matching is

$$\frac{R_S R_{SA}}{R_S + 2R_{SA}} = \frac{Z_A Z_B}{Z_A + 2Z_B} = \frac{R_L R_{LA}}{R_L + 2R_{LA}} \tag{2.41}$$

Two possible simultaneous conditions for correct matching are therefore:

$$R_S = Z_A = R_L \tag{2.42}$$

and

$$R_{SA} = Z_B = R_{LA} \tag{2.43}$$

We have seen that, ultimately for the best EMC resilient interfacing, we aim for the differential connections shown in Figure 3-3. This is very close to what we have in Figure 3-4 if the resistors connected to ground R_{SA} and R_{LA} were not present (or each of infinite resistance). (2.41) may be re-written as:

$$\frac{R_S}{R_S/R_{SA} + 2} = \frac{Z_A Z_B}{Z_A + 2Z_B} = \frac{R_L}{R_L/R_{LA} + 2} \tag{2.44}$$

Making R_{SA} and R_{LA} very large with respect to R_S and R_L respectively then

$$R_S = \frac{2Z_A Z_B}{Z_A + 2Z_B} = R_L \tag{2.45}$$

We have seen this before in Figure 3-3. Clearly therefore for differential excitation we would wish R_{SA} and R_{LA} to be as large as possible.

3.3.2 Perfectly *Common Mode* Excitation

Previously we have concentrated on (perfect) differential excitation of balanced lines and loads because of their attractive EMC properties: suppression of radiated noise and resilience to radiated interference from elsewhere. *Common mode* excitation requires ground connections, perhaps via a ground plane, to allow the return current to flow, so this is only possible with architectures which include ground connections. For example, those shown in Figure 3-2 and Figure 3-3 could not support common mode excitation because there are no ground paths. Indeed, that is the chief advantage of these types, if they could be designed reliably and it is rare that we intentionally use common mode excitation. Some level of common mode excitation is usually the result of design limitations. We will take a similar approach with perfect common mode excitation as we did with perfect differential mode excitation, starting with the architecture shown in Figure 3-9.

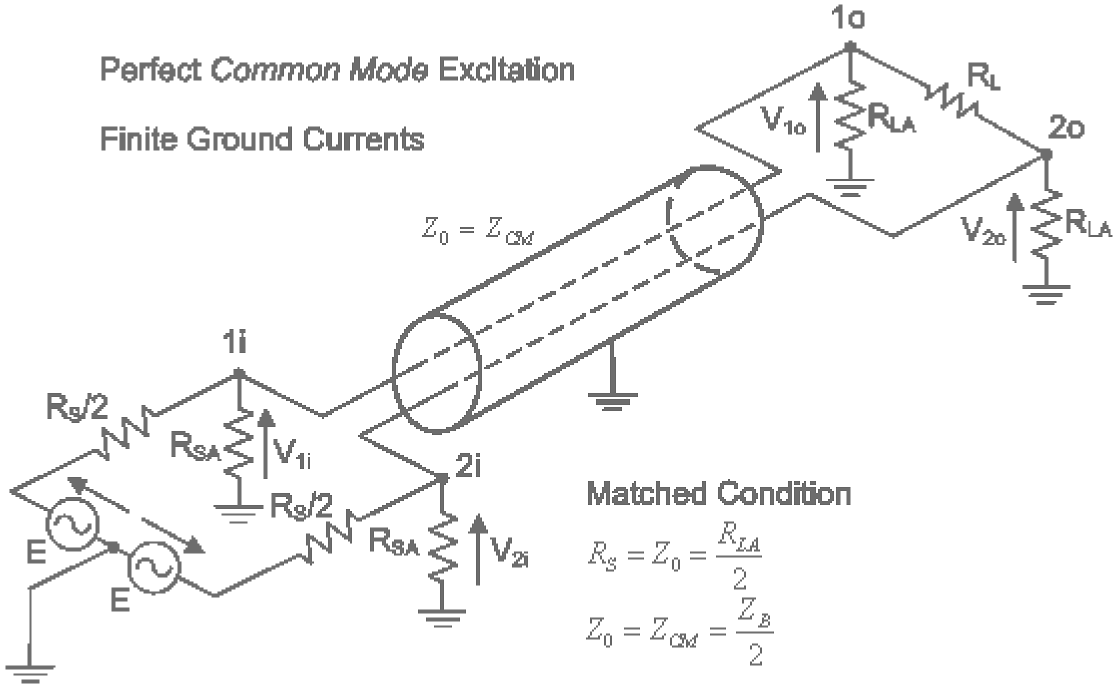


Figure 3-9 Connection of a common mode excited balanced source to a balanced load using a screened, balanced pair transmission line. Further resistors R_{SA} and R_{LA} represent typical realistic loading conditions

This is very similar to the configuration shown in Figure 3-4 except that now both of the voltage sources are in phase. Therefore the voltages at the TL input conductors 1i and 2i, say V_{1i} and V_{2i} respectively, are both identical magnitude and phase. (In the differentially excited case the corresponding voltages were in anti-phase). Figure 3-10 shows the first stage equivalent circuit in which two simplifications are clear.

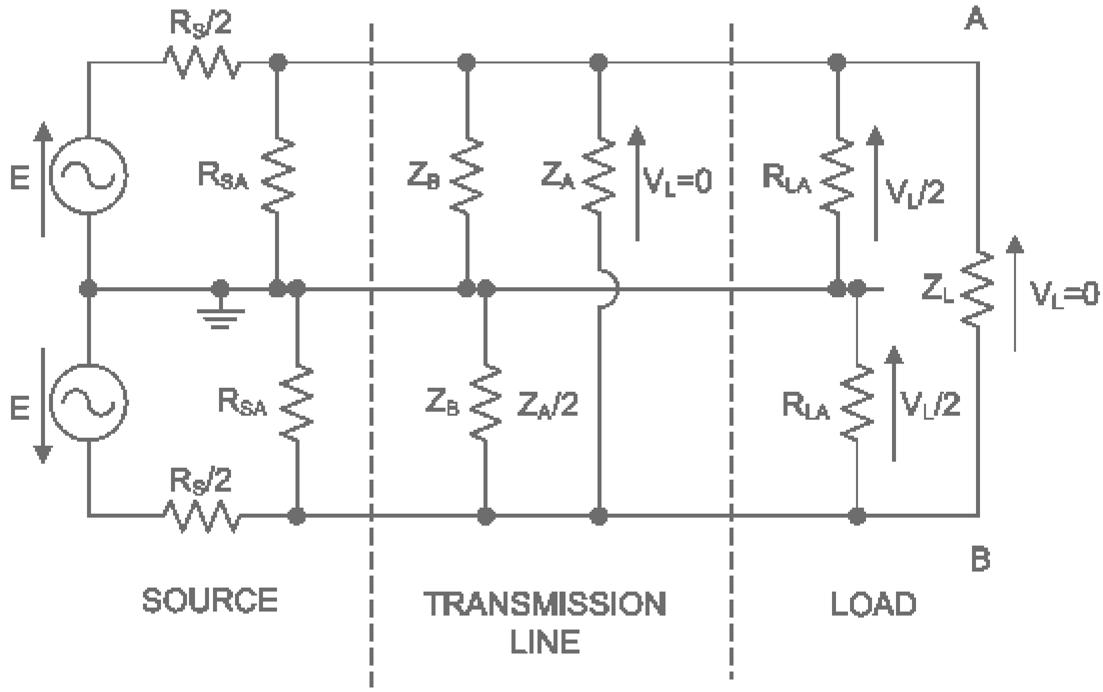


Figure 3-10 The first stage equivalent circuit for perfectly common mode excitation

From the symmetry of the balanced circuit, it is clear that the voltages at A and B will be identical. Therefore there will be no current flow through any components connected between these points, in this case Z_A and Z_L. Figure 3-11 shows the simplified equivalent circuit with these components removed and the equivalent voltage sources simplified as shown in Figure 3-7.

Therefore the conditions for best match are:

$$\frac{R_S R_{SA}}{R_S + 2R_{SA}} = Z_B = R_{LA} \tag{2.46}$$

Of course, with perfectly common mode excitation of a circuit that is designed for differential mode operation, there will be no power dissipated in the differential load Z_L.

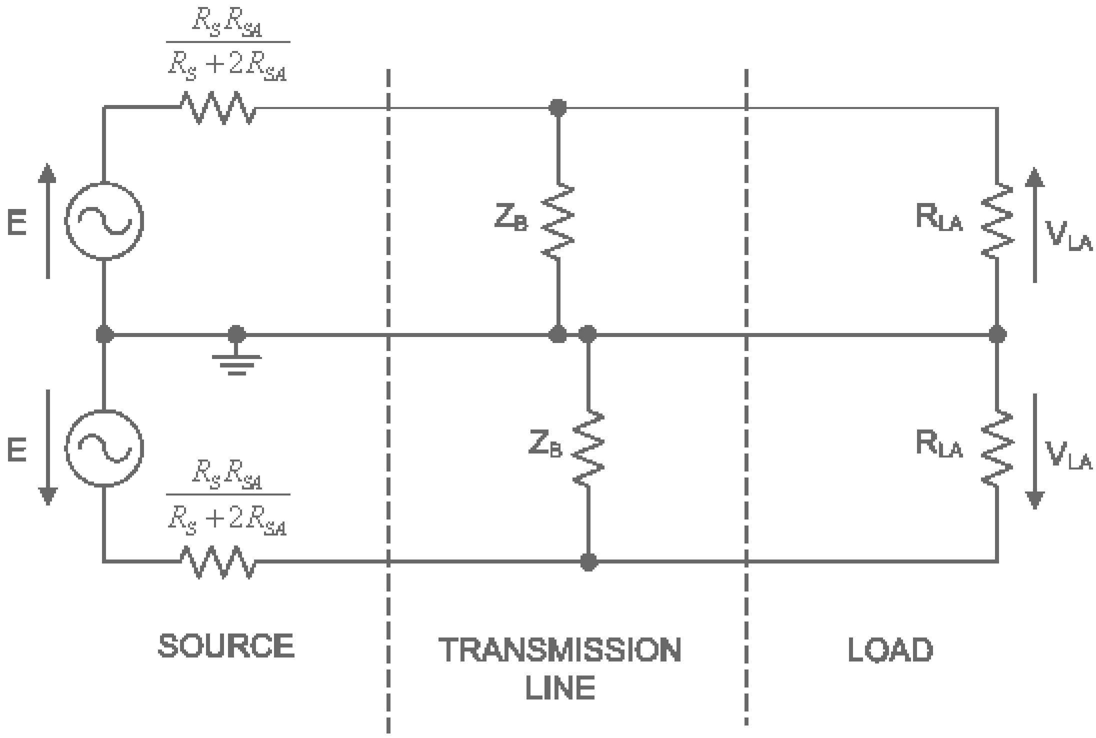


Figure 3-11 The simplified equivalent circuit for perfectly common mode excitation

4. ARBITARY EXCITATION OF BALANCED LINES AND LOADS

We have looked particularly at the *perfect differential excitation* of balanced transmission lines feeding balanced loads due to their attractive EMC properties. When designing practical high speed interfaces, we normally *aim to achieve* these conditions but inevitably the result is often less than ideal: some common mode will be present.

Figure 4-1 shows the circuit we will use to analyse the voltages and currents for the generic case, where there could be finite levels of differential mode and/or common mode signals present. In particular, note the directions of the chosen currents and voltages. The analysis will be performed using Ohm's Law and Kirchoff's Current Circulation laws.

The left side comprises two Thevenin equivalent AC constant voltage sources each operating at the identical frequency f hertz (Hz), each with equivalent source resistance R_S. The balanced load comprises a differential resistor R_d and two single ended (common mode) resistors R_C connected as shown. All resistor values are in ohms (Ω).

The voltage sources are labelled in complex exponential format with instantaneous (magnitude and phase) voltages $E_1(t)$ and $E_2(t)$ given by:

$$E_1(t) = E_{10}e^{j2\pi ft} \quad (3.1)$$

for the top source and

$$E_2(t) = E_{20}e^{j(2\pi ft - \Delta\phi)} \quad (3.2)$$

for the bottom source.

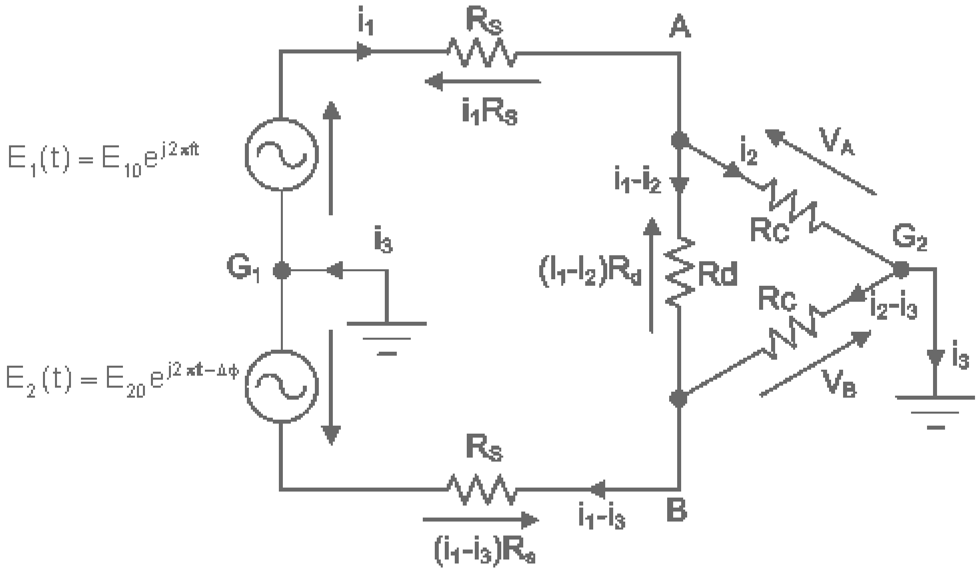


Figure 4-1 The equivalent circuit using generic symmetric sources running at identical frequencies, arbitrary phase relationships and a symmetric load about the ground connection

The voltage *magnitudes* for $E_1(t)$ and $E_2(t)$ are E_{10} and E_{20} respectively and the instantaneous time (t) is in seconds (s). The (constant) phase angle of E_2 relative to E_1 is $-\Delta\phi$ radian (rad).

Using Kirchoff's and Ohm's laws, the circuit may be resolved into loop equations as follows:

- Loop G_1ABG_1 :

$$E_1(t) - E_2(t) = i_1(2R_s + R_d) - i_2R_d - i_3R_s \tag{3.3}$$

- Loop $G_1AG_2G_1$:

$$E_1(t) = i_1R_s + i_2R_c \tag{3.4}$$

- Loop AG_2BA

$$0 = i_1R_d - i_2(2R_c + R_d) + i_3R_c \tag{3.5}$$

These equations may be expressed in a matrix form thus:

$$\begin{pmatrix} 2R_s + R_d & -R_d & -R_s \\ R_s & R_c & 0 \\ R_d & -\ 2R_c + R_d & Rc \end{pmatrix} \begin{pmatrix} i_1 \\ i_2 \\ i_3 \end{pmatrix} = \begin{pmatrix} E_1(t) - E_2(t) \\ E_1(t) \\ 0 \end{pmatrix} \tag{3.6}$$

For clarity, we will represent the respective matrices in (3.6) by the following simpler expressions in which matrix variables are represented by the symbols enclosed in round brackets:

So:

$$R = \begin{pmatrix} 2R_s + R_d & -R_d & -R_s \\ R_s & R_c & 0 \\ R_d & -\ 2R_c + R_d & R_c \end{pmatrix} \tag{3.7}$$

$$i = \begin{pmatrix} i_1 \\ i_2 \\ i_3 \end{pmatrix} \tag{3.8}$$

and

$$E = \begin{pmatrix} E_1(t) - E_2(t) \\ E_1(t) \\ 0 \end{pmatrix} \tag{3.9}$$

Therefore:

$$R \quad i = E \tag{3.10}$$

and, by the rules of matrix algebra:

$$R^{-1} \ E = i \tag{3.11}$$

In (3.11) R^{-1} means 'the matrix inverse of R'. Calculating the inverse of (3.7) is somewhat laborious, but helped with one of the 'math' type applications with symbolic capability. In this case, MathCad 15 was used [10]. This gave the following result:

$$R^{-1} = \begin{pmatrix} \dfrac{R_c^{\,2}}{R_c + R_s \quad R_cR_d + 2R_cR_s + R_dR_s} & \dfrac{1}{R_c + R_s} & \dfrac{R_cR_s}{R_c + R_s \quad R_cR_d + 2R_cR_s + R_dR_s} \\ \dfrac{R_cR_s}{R_c + R_s \quad R_cR_d + 2R_cR_s + R_dR_s} & \dfrac{1}{R_c + R_s} & \dfrac{R_s}{R_c + R_s \quad R_cR_d + 2R_cR_s + R_dR_s} \\ -\dfrac{1}{R_c + R_s} & \dfrac{2}{R_c + R_s} & \dfrac{1}{R_c + R_s} \end{pmatrix} \tag{3.12}$$

Using this result, the values for i_1, i_2 and i_3 may be obtained from the following matrix equation.

$$i = R^{-1} E$$

$$= \begin{pmatrix} \dfrac{R_c^2 \;\; E_1(t) - E_2(t)}{R_c + R_s \quad R_d R_c + 2R_c R_s + R_d R_s} + \dfrac{E_1 \;\; t}{R_c + R_s} \\ \dfrac{R_d R_c + R_c R_s + R_d R_s \;\; E_1(t) + R_c R_s E_2(t)}{R_c + R_s \quad R_c R_d + 2R_c R_s + R_d R_s} \\ \dfrac{E_1(t) + E_2(t)}{R_c + R_s} \end{pmatrix} \tag{3.13}$$

Relating (3.13) to (3.8) therefore we have the following three equations:

$$i_1 = \frac{E_1 \;\; t}{R_c + R_s} + \frac{R_c^2 \;\; E_1(t) - E_2(t)}{R_c + R_s \quad R_d R_c + 2R_c R_s + R_d R_s} \tag{3.14}$$

$$i_2 = \frac{E_1 \;\; t}{R_c + R_s} - \frac{R_c R_s \;\; E_1(t) - E_2(t)}{R_c + R_s \quad R_d R_c + 2R_c R_s + R_d R_s} \tag{3.15}$$

$$i_3 = \frac{E_1(t) + E_2(t)}{R_c + R_s} \tag{3.16}$$

These are general expressions which apply whatever the level of differential and/or common mode excitation. Now in the following sections we will use each of these current expressions in the two perfect modes of excitation, differential and common.

4.1 Perfectly Differential Mode Excitation

Referring to Figure 4-1, for perfectly differential excitation, the voltage sources $E_1(t)$ and $E_2(t)$ must:

- have identical voltage magnitudes ($E_{10} = E_{20}$);
- have a phase relationship of 180º (or $\pi\ rad$) so, from (3.2), $\Delta\phi = \pi$.

To analyse this further we need to substitute for $E_1(t)$ and $E_2(t)$ from (3.1) and (3.2) respectively, into the expression for i_3 in (3.16), the result being:

$$i_3 = \frac{E_{10} e^{j2\pi ft} + E_{20} e^{j(2\pi ft - \Delta\phi)}}{R_c + R_s} = \frac{E_{10} e^{j2\pi ft} + E_{20} e^{j2\pi ft} e^{-j\Delta\phi}}{R_c + R_s} \tag{3.17}$$

In complex exponentials it is never long before we need to call upon another of Euler's equations, and this is no exception. Here we will use the following for conversion from complex exponentials to complex trigonometric functions.

$$e^{-jx} = \cos x - j\sin x \tag{3.18}$$

Therefore:

$$\begin{aligned} e^{-j\Delta\phi} &= \cos \Delta\phi - j\sin \Delta\phi \\ &= \cos\pi - j\sin\pi \\ &= -1 - j.0 \\ &= -1 \end{aligned} \tag{3.19}$$

So for the perfectly differential case:

$$i_3 = \frac{e^{j2\pi ft}(E_{10} - E_{20})}{R_s} \tag{3.20}$$

The other condition for equal voltage magnitudes is $E_{10} = E_{20}$, so

$$i_3 = 0 \tag{3.21}$$

So, for perfectly differential excitation, there is no current flow through the ground connections. Therefore $i_3 = 0$ and

$$\begin{aligned} V_A = V_B &= \frac{R_c \ R_cR_dE_1(t) + R_cR_sE_1(t) + R_dR_sE_1(t) + R_cR_sE_2(t)}{(R_c + R_s) \ R_cR_d + 2R_cR_s + R_dR_s} \\ &= \frac{R_c\left[R_cR_d + R_cR_s + R_dR_s \ E_1(t) + R_cR_sE_2(t)\right]}{(R_c + R_s) \ R_cR_d + 2R_cR_s + R_dR_s} \end{aligned} \tag{3.22}$$

The voltage between A and B, V_{AB} is

$$\begin{aligned} V_{AB} &= 2V_A = 2V_B \\ &= \frac{2R_c\left[R_cR_d + R_cR_s + R_dR_s \ E_1(t) + R_cR_sE_2(t)\right]}{(R_c + R_s) \ R_cR_d + 2R_cR_s + R_dR_s} \\ &= \frac{2R_c\left[R_cR_d + R_cR_s + R_dR_s \ E_{10}e^{j2\pi ft} + R_cR_sE_{20}e^{j2\pi ft}e^{-j\Delta\phi}\right]}{(R_c + R_s) \ R_cR_d + 2R_cR_s + R_dR_s} \\ &= \frac{2R_c\left[R_cR_d + R_cR_s + R_dR_s \ E_{10}e^{j2\pi ft} - R_cR_sE_{10}e^{j2\pi ft}\right]}{(R_c + R_s) \left(R_cR_d + 2R_cR_s + R_dR_s \right)} \end{aligned} \tag{3.23}$$

The expression for V_{AB} simplifies after allowing for the conditions necessary for perfectly differential excitation noted at the beginning of this section ($\Delta\phi = \pi \quad rad$ and $E_{10} = E_{20}$), therefore

$$V_{AB} = \frac{2R_c R_d E_{10}{}^{j2\pi ft}}{R_c R_d + R_s(2R_c + R_d)}$$
$$= \frac{2R_c R_d E_{20}{}^{j2\pi ft}}{R_c R_d + R_s(2R_c + R_d)} \qquad (3.24)$$

As we noted, for perfectly differential excitation, the voltage sources are both at the identical frequency and are exactly in anti-phase. Therefore, the voltage sources can be considered to be DC but with opposite signs, measured relative to ground. In Figure 4-2 therefore, the voltage at one source is $+E$ whilst the voltage at the other source is $-E$.

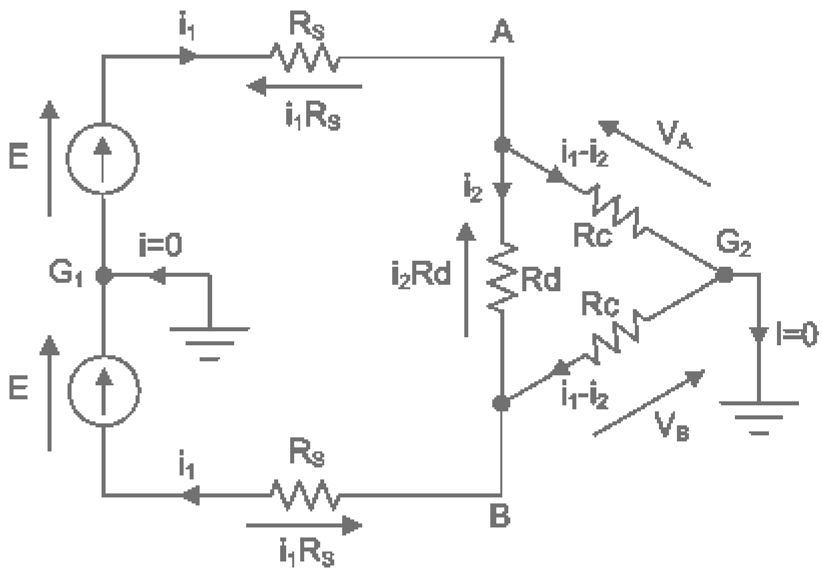


Figure 4-2 The equivalent DC circuit schematic for perfectly differential excitation

We know that the ground current is zero so the load presented between nodes A and B, say R_L is given by

$$R_L = \frac{2R_c R_d}{R_d + 2R_c} \qquad (3.25)$$

By the potentiometer effect

$$V_{AB} = \frac{2R_L E}{R_L + 2R_s}$$
$$= \frac{2R_d R_c E}{2R_d R_c + R_s \;\; R_d + 2R_c} \qquad (3.26)$$

4.2 Perfectly Common Mode Excitation

Referring to Figure 4-1, for perfectly common mode excitation, the voltage sources $E_1(t)$ and $E_2(t)$ must:

- have identical voltage magnitudes ($E_{10} = E_{20}$);
- have identical phase so, from (3.2), $\Delta\phi = 0$.

The voltage between nodes A and B, V_{AB} is given by

$$V_{AB} = (i_1 - i_2) R_d \tag{3.27}$$

Substituting for i_1 and i_2 from (3.14) and (3.15) respectively gives:

$$\begin{aligned} V_{AB} &= (i_1 - i_2) R_d \\ &= \frac{R_c R_d (E_1(t) - E_2(t))}{R_c R_d + R_s (2R_c + R_d)} \end{aligned} \tag{3.28}$$

From (3.1) and (3.2) it is clear that, under the conditions of perfectly *common mode* excitation, for which $E_{10} = E_{20}$ and $\Delta\phi = 0$, then $E_1(t) = E_2(t)$ and $V_{AB} = 0$.

Since $V_{AB} = 0$, then there must be no current through the resistor R_d, so

$$\begin{aligned} i_1 - i_2 &= 0 \\ i_1 &= i_2 \end{aligned} \tag{3.29}$$

Noting the voltage directions defined for V_A and V_B are opposite so, since $V_{AB} = 0$,

$$V_A = -V_B \tag{3.30}$$

Therefore

$$\begin{aligned} i_2 R_c &= -(i_2 - i_3) R_c \\ 2i_2 &= i_3 \end{aligned} \tag{3.31}$$

The current through the lower source resistor R_s, $i_1 - i_3$ therefore has the same magnitude as i_1 but is *in the opposite direction.*

For perfectly common mode excitation, the voltage sources are both at the identical frequency and are exactly in phase. Therefore, the voltage sources can be considered to be

DC of identical signs, measured relative to ground as shown in Figure 4-3. Therefore, the voltages at both sources are simultaneously $+E$.

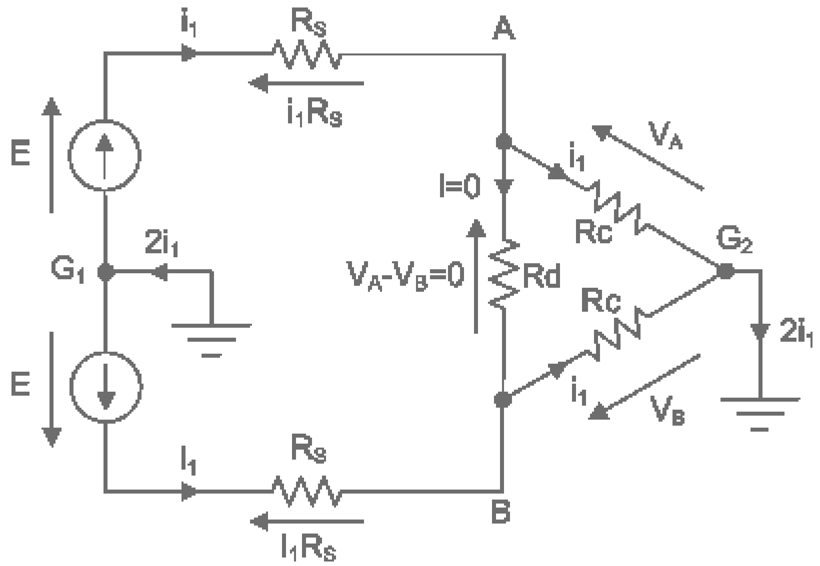


Figure 4-3 The equivalent DC circuit schematic for perfectly common mode excitation

5. REFERENCES

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